Combined Power Control and Transmission Rate Selection in Cellular Networks

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Abstract— **Emerging multimedia services in cellular radio systems introduce a *variable* transmission rate, which raises the problem of controlling transmission rates in a spectrally efficient way. In this paper, we formulate the problem into a combined power and rate control, for which we suggest two different algorithms. In the first one, we derive an algorithm applying the *Lagrangian relaxation* technique. In the other method, called *selective power control*, we extend a *fixed* rate power control algorithm to solve the problem. Computational experiments carried out on a CDMA system indicate that the proposed algorithms give satisfying performance in terms of system throughput, outage probability and transmission power consumption.**

I. Introduction

Next-generation cellular radio systems will be able to provide multimedia services, which are characterized by different QoS requirements such as minimum transmission rates. For a real time service, users must be guaranteed a tolerable minimum rate. However, non-real time applications (delay insensitive) may temporally lower their transmission rates even to zero, utilizing any excess capacity that the radio network can provide in a *best effort fashion* [1]. The availability of *variable* transmission rates in a radio network raises the problem of controlling them in the most spectrally efficient way. In the radio channel, transmission rates are closely related to signal-to-interference ratios (SIRs), and the SIRs can be efficiently controlled by power control. Therefore, it is natural to associate rate control with power control, which will be investigated in this paper.

There has been a substantial amount of work on efficient power controls for the *fixed* rate system. However, very few studies have suggested power control algorithms that support variable transmission rates efficiently. In [2], maximizing the total transmission rate is considered in the context of a CDMA system. A more general formulation is provided in [3], where a combined problem of base station assignment, rate control and power control in a CDMA system is formulated into a *quadratic programming problem*. The recent paper [4] addresses the problem in terms of joint power control and *adaptive* modulation, in which powers are controlled within lower and upper limits to maximize the *log-sum* of users' SIR values. The underlying idea is that, by maximizing the SIR values, we also maximize the total transmission rate through adaptive modulation techniques. The models in [1]-[4] assume that feasible transmission rates may take any continuous value; the throughput of a radio channel is a *continuous* function of channel quality. The continuity assumption in these papers greatly simplifies the problem. However, the resulted formulations have *nonlinear* terms either in the objective function or in the constraints, which in turn do not lend themselves into *distributed* power control algorithms. Note that in practice, the feasible transmission rates are limited to a small number of *discrete* values. With this in mind, we will focus our attention on the rate selection and power control problem when the feasible transmission rates are discrete. The problem is formally presented in Section II, where we formulate the combined rate and power control problem as a *linear optimization model.*

We apply two different methods to solve the problem. In Section III, based on the mathematical formulation of Section II, we derive a distributed algorithm that finds a sub-optimal solution by utilizing the *Lagrangian relaxation technique* [8] which has been also successfully applied to *transmitter removal* [5]. In Section IV, we suggest another algorithm, called *selective power control* by exploiting a fixed rate power control algorithm [6]. Our algorithms are fully distributed and use only local SIR measurement and signaling. Finally, the algorithms are evaluated in Section V, where we compare between them in a CDMA system that supports different data rates using the variable processing gain. The result shows that the Lagrangian relaxation based algorithm is favorable when the system throughput is a major concern. However, the selective power control is much more advantageous in terms of *energy efficiency* defined as the data rate supported by a certain amount of energy.

II. System Model

Consider a frequency channel in a cellular radio system with N mobiles accessing the channel. Without loss of generality, we will consider only uplink throughout this paper. Each mobile i ($1 \leq i \leq N$) can transmit with power $0 \leq p_i \leq \overline{p}_i$, where $\overline{p}_i$ is its maximum transmission power.

We consider a short time interval such that the link gain between every mobile j and every base station i is stationary and is given by g_{ij} $(1 \le i, j \le N)$. We define $a_i = j$ if mobile i is assigned to base station j. Base station assignment can be represented by a vector $A = (a_1, a_2, \ldots, a_N)$. Given a *power vector* $P = (p_1, p_2, \ldots, p_N)$, the received signal-to-interference-plus-noise ratio of mobile i, is defined by

$$SIR_i(P) \stackrel{\text{def}}{=} \frac{g_{a_i i}\ p_i}{\nu_{a_i} + \sum_{j:j\neq i} g_{a_i j}\ p_j} \qquad (1 \le i \le N), \quad (1)$$

where $\nu_{a_i} > 0$ is the background noise power at base station a_i.

The feasible data rate of a user depends on many factors such as receiver structure, user mobility, etc. Nevertheless, the SIR is a reasonable measure to evaluate the effective data rates. For any given physical layer technique, let $r_i^1 < r_i^2 < \ldots < r_i^K$, be the rates that mobile i can utilize. For the sake of readability, we assume that all mobiles have the same set of feasible data rates. To properly receive messages at transmission rate r_i^k, mobile i is expected to attain $SIR_i(P) \ge \gamma_i^k$.

Let $Y = [y_i^k]$ be a 0-1 matrix such that, for every mobile i and rate r_i^k,

$$y_i^k = \begin{cases} 1, & \text{if mobile } i \text{ is transmitting with rate } r_i^k, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Further, let M_i^k be an arbitrarily large number satisfying

$$M_i^k \ge \max_P \frac{p_i \cdot \gamma_i^k}{SIR_i(P)}. \quad (3)$$

By the definition of (1), the value M_i^k can be interpreted as the amount of transmission power that mobile i needs to attain γ_i^k, regardless of the interference power. Then, the combined rate and power control is formulated as the following optimization problem:

$$W \stackrel{\text{def}}{=} \max_{Y,P} \sum_{i=1}^{N}\sum_{k=1}^{K} w_i^k\ y_i^k, \quad (4)$$

subject to the constraints that for every i and k,

$$\sum_{k=1}^{K} y_i^k \le 1,\ y_i^k \in \{0,1\}, \text{ and } 0 \le p_i \le \overline{p}_i, \quad (5)$$

$$p_i + (1 - y_i^k)M_i^k \ge \frac{p_i \cdot \gamma_i^k}{SIR_i(P)}. \quad (6)$$

To associate rewards to effective (realized) transmission rates, in the objective function (4), we assume that so long as mobile i is properly transmitting messages at rate r_i^k, the system accrues a unit reward with rate w_i^k. Here we will use $w_i^k = r_i^k$, aiming at the maximization of the instant system throughput, i.e., the sum of effective data rates of all users at the given instant.

Observe that the controlled powers are not incorporated into the objective function; their sole role is to facilitate proper demodulation under a selected set of rates. This is mathematically expressed in constraint (6). For each i and k, if $y_i^k = 1$, then the constraint (6) is reduced to the SIR requirement on mobile i for rate r_i^k. Setting M_i^k as in (3), neutralizes the constraint (6) for non-selected rates with $y_i^k = 0$. Even if we have not considered the powers in the objective function, the amount of data that can be reliably sent using a certain energy allocation may be critical. This will be discussed in Section V.

III. Power Control Using Lagrangian Multipliers

For given Y and P, define

$$d_i^k(Y,P) \stackrel{\text{def}}{=} \left[p_i + (1 - y_i^k)M_i^k - \frac{p_i \cdot \gamma_i^k}{SIR_i(P)}\right] \quad (7)$$

And, for a given nonnegative matrix of *Lagrangian multipliers* $\Lambda = [\lambda_i^k]$, consider the following *Lagrangian relaxation problem* [8],

$$L(\Lambda) \stackrel{\text{def}}{=} \max_{Y,P} \left\{\sum_{i=1}^{N}\sum_{k=1}^{K} y_i^k\ w_i^k + \sum_{i=1}^{N}\sum_{k=1}^{K} \lambda_i^k \cdot d_i^k(Y,P)\right\}. \quad (8)$$

subject to the constraints (5).

The above problem is constructed as follows: The constraint (6) of the *original problem* is multiplied by the given Λ, and then incorporated into the objective function of the original problem. It is easy to show that $L(\Lambda) \ge W$ for any $\Lambda \ge 0$. The difference $L(\Lambda) - W$ is often called the *Lagrangian duality gap* [8]. When the duality gap is sufficiently small, then the Lagrangian relaxed problem with Λ is considered as a *good* approximation of the original problem. If the Lagrangian duality gap is small enough, then it is very probable that the optimal solution of the relaxed problem, say $\hat{Y}$ and $\hat{P}$, is close to that of the original problem.

We propose an iterative algorithm that updates the *target* rate (or equivalently *target* SIR) for each mobile in every iteration, by resolving $\hat{Y}$ from the Lagrangian relaxation problem with a given set of Lagrangian multipliers. Since the multipliers are also updated in each iteration, the target rate of each mobile will be changed from one iteration to another. For a given Λ, let $\hat{Y}(\Lambda) = [\hat{y}_i^k(\Lambda)]$ be the optimal solution to the Lagrangian relaxation problem. Then, we can derive that the optimal solution $\hat{Y}(\Lambda)$ is obtained by

$$\hat{y}_i^k(\Lambda) = \begin{cases} 1, & \text{if } w_i^k - \lambda_i^k M_i^k = \max_m\{w_i^m - \lambda_i^m M_i^m\} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

In the above, if the maximum is attained by more than one value, then the one with the largest k is selected. That is, the target rate r_i^k selected by mobile i is the largest among the ones fulfilling $w_i^k - \lambda_i^k M_i^k = \max_m\{w_i^m - \lambda_i^m M_i^m\}$, provided the value $w_i^k - \lambda_i^k M_i^k$ is positive. Otherwise, mobile i does not transmit. Note that the only local information s used to solve $\hat{y}_i^k(\Lambda)$, since the values w_i^k, M_i^k and λ_i^k are ssumed to be known to mobile i. Given the target rate as letermined by $\hat{y}_i^k(\Lambda)$, the target SIR for each mobile is the me that corresponds to the target rate. With that target SIR, the transmission power is then updated by executing DCPC [7] for one step. The DCPC update procedure is lefined below in our proposed algorithm.

Now, let $\Lambda(n) = [\lambda_i^k(n)]$ and $P(n) = (p_i(n))$ be respectively the set of Lagrangian multipliers and the power vector at iteration n. Then, our algorithm is defined as follows:

LRPC

0. *Initialize*:
   Set the iteration number $n = 1$, and fix $\Lambda(n)$ and $P(n)$.
1. *Select rates*:
   1.1. For every mobile i, solve $\hat{y}_i^k(\Lambda(n))$ from the equation (9).
   1.2. If $\hat{y}_i^k(\Lambda(n)) = 0$ for all k, then mobile i is not transmitting during iteration n. Otherwise, mobile i uses the target γ_i^k that corresponds to the rate r_i^k with $\hat{y}_i^k(\Lambda(n)) = 1$.
2. *Power update*:
   For every mobile i, execute DCPC by one step with the target γ_i^k that has been selected in the Step 1.2. That is, update the power by

$$p_i(n+1) = \min\left\{\frac{p_i(n)\cdot\gamma_i^k}{SIR_i(P(n))},\ \overline{p_i}\right\}. \qquad (10)$$

3. *Lagrangian multiplier update*:
   For every mobile i, update $\lambda_i^k(n)$ by the modified *subgradient method* given by

$$\lambda_i^k(n+1) = \max\left\{0, \lambda_i^k(n) - \frac{d_i^k\left(\hat{Y}(\Lambda(n)), P(n)\right)}{n}\right\}, \qquad (11)$$

   where the definition of d_i^k is given in (7).
4. *Continue*:
   When a stopping criterion is met, stop. Otherwise, set $n = n + 1$ and go to Step 1.

More detailed description about how the subgradient method [9] was modified in Step 3 is necessary but we only described our update procedure. Observe that each step in the algorithm uses only local information and measurement that is available in each mobile i.

IV. Selective Power Control

In contrast to LRPC in the previous section, the algorithm proposed here does not directly rely upon the optimization formulation of (4)-(6). The algorithm tries to maximize the sum of effective data rates with the minimal energy consumption. It is independent of the reward rates $\{w_i^k\}$, which may not be an obstacle as reward rates are hard to specify in practice. The algorithm is motivated from a fixed rate power control algorithm recently suggested in [6], and described as follows:

Selective Power Control (SPC)

For any feasible power vector $P(n)$, the next transmission power used by mobile i is determined by:

$$p_i(n+1) = \max_k\left\{\frac{p_i(n)\cdot\gamma_i^k}{SIR_i(P(n))}\cdot\chi\left(\frac{p_i(n)\cdot\gamma_i^k}{SIR_i(P(n))} \le \overline{p}_i\right)\right\}, \qquad (12)$$

where $\chi(E)$ is the indicator function of the event E.

SPC attempts to utilize the maximum feasible rate at any iteration, assuming other users keep current power levels. However, it improves the energy efficiency, by trying not to increase the mobile power beyond a power that can be translated into a feasible rate. If the mobile cannot be supported even with its minimum rate within the power range, it does not transmit.

V. Computational Experiments

The main purpose of the experiments is to draw insight on how LRPC and SPC perform in a multirate system. To put the performance of our proposed algorithms in a wider prospective, we also compare them with a multirate power control algorithm [4]. The convergent point of the algorithm is optimal in the sense that it maximizes $\prod_{i=1}^{N} SIR_i(P)$. However, since the algorithm requires link gains for power update, we will call it CPC (centralized multirate power control)

The testbed is a CDMA system with 19 omni-bases located in the centers of two-tired 19 hexagonal cells. The distance between two nearest base stations is 2 km. We consider the uplink of the system that has a chip rate of 1.2288 Mcps and assume that the radio link can support four data rates, $r_i^k = 9.6 \cdot \frac{1}{2^{k-1}}$ kbps ($k = 1, 2, 3, 4$). At any given instance, a total of 190 mobiles are generated and are uniformly distributed over the entire 19 cells. The link gain, g_{ij} is modeled by $g_{ij} = s_{ij} \cdot d_{ij}^{-4}$, where s_{ij} is the shadow fading factor and d_{ij} is the distance between base i and mobile j. The variables s_{ij}'s are assumed to be independently and identically distributed log-normal random variables, with mean of 0 dB and standard deviation of 6 dB. The base receiver noise is taken to be -150 dB, and the maximum mobile power is set to 0 dB. At each instance, the initial transmission power of each mobile is randomly chosen from the interval [0,1], and each mobile is connected to the base station that provides the lowest attenuation. We consider the *single-code* system in which multiple rates are realized by the variable processing gain that is defined as the ratio of chip rate to the user information bit rate. The required minimum SIR before *despreading* is assumed to be $\gamma_i^k = \gamma \cdot \frac{1}{2^{k-1}}$ for each $9.6 \cdot \frac{1}{2^{k-1}}$ kbps. Three values, -14 dB, -10 dB and -6 dB are considered for γ, representing light, medium and heavy loads, respectively. The required E_b/I_o, *bit energy-to-interference power spectral density*, is calculated by adding the processing gain to the corresponding SIR value (in dB), which is constant, regardless of target data rates. For example, when $\gamma = -14$ dB, the required E_b/I_o is about 7 dB

The *average effective rate per mobile*, the *outage probability* and the *average transmission power per mobile* are used as performance measures. To evaluate them, we have taken 100 independent instances of mobile locations and shadow fadings. In each instance, we have performed the three algorithms, LRPC, SPC and CPC, until the iteration number, n reaches thirty. In every iteration n, the *realized* effective transmission rate of mobile i is defined by $r_i(n) = r^{k^*}$, where the index k^* is

$$k^* \stackrel{\text{def}}{=} \max\left\{k : SIR_i(P(n)) \geq \gamma_i^k\right\}. \qquad (13)$$

The outage probability at each iteration is evaluated by counting the proportion of mobiles that cannot even attain the *minimum rate* 1.2 kbps.

In Figures 1-3, we present the outage probability of each algorithm for the three load scenarios, as a function of iteration number. It can be observed that there is no significant difference among three algorithms under the light load, making all users supported at least with the minimum rate (Figure 1). However, SPC yields a slightly lower outage than LRPC in the heavy load scenario (Figure 3). It is noticeable that CPC stably achieves the lowest outage among the algorithms. Especially in the heavy load scenario, it gives a significantly low outage (Figure 3). This is explained by the fact that CPC attempts to *balance* the radio resource to guarantee the minimum rate to each mobile. In fact, CPC does not set the power value of any user to zero even under heavily loaded situations, due to its intrinsic feature of maximizing $\prod_{i=1}^{N} SIR_i(P)$.

Figures 4-6 depict the average effective rate that is obtained by each algorithm in the different load scenarios. Note that the total throughput is proportional to the area below each algorithm's curve. The results indicate that CPC has the lowest throughput across all cases. However, LRPC gives steadily over the iterations the best performance. LRPC results in slightly lower throughput than SPC in the light load (Figure 4), whereas SPC converges to the state that gives the maximum throughput, 9.6 kbps per mobile. SPC is not steady (i.e, oscillating between the data rates) in the medium load scenario (Figure 5), but provides almost the same best throughput as LRPC in the heavy load (Figure 6). In Figures 7-9, we further investigate energy consumption levels of each algorithm. As can be noticed, CPC gives the same performance under the different load conditions. It is because CPC converges to the same fixed point, regardless of the target SIR (rate). SPC consumes much less power than LRPC as the load increases. Considering the throughput of SPC in the two extreme loads (Figures 4 and 6), we suggest SPC rather than LRPC in both light and heavy load conditions.

Summarizing the results of Figures 1-9, we may conclude that if the throughput is a major concern, then LRPC is practically best across all load scenarios. However, when it comes to the energy efficiency defined as the effective data rate supported by a certain amount of energy, SPC is a strong alternative under the light and the heavy loads.

VI. Concluding Remarks

In this paper, we studied the combined rate and power control for a cellular radio system that can support different transmission rates in a single connection. The problem is formulated into a manageable linear optimization model, for which we derived two different fully *distributed* algorithms, LRPC and SPC. Computational experiments carried on a CDMA system indicate that both algorithms significantly improve the system throughput. Finally, we would like to mention that the system throughput is strongly dependent on the effective rate determination procedure as was defined in (13), which in turn relies on the receiver structure. Therefore, further study on performance difference between different receivers is an interesting future research topic.

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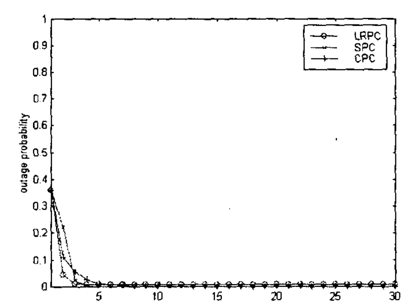

Fig. 1. Outage probability under light load.

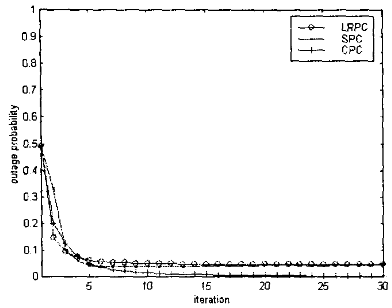

Fig. 2. Outage probability under medium load.

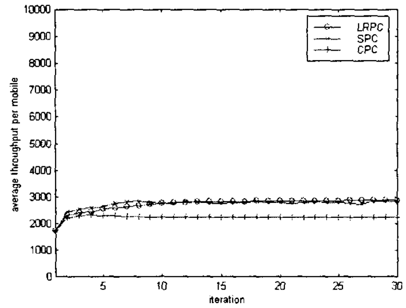

Fig. 6. Average effective rate per mobile under heavy load.

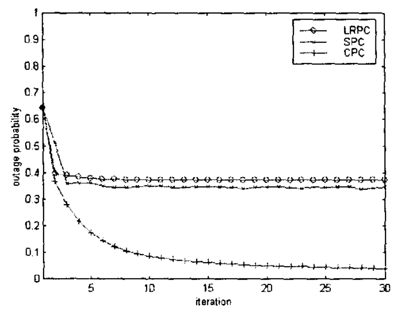

Fig. 3. Outage probability under heavy load.

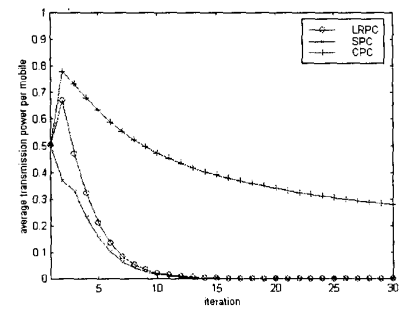

Fig. 7. Average transmission power per mobile under light load.

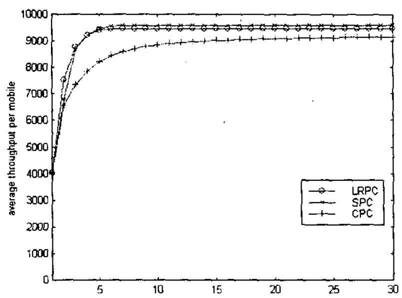

Fig. 4. Average effective rate per mobile under light load.

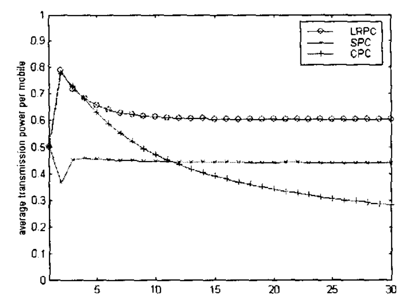

Fig. 8. Average transmission power per mobile under medium load.

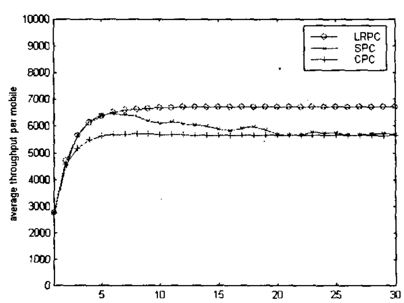

Fig. 5. Average effective rate per mobile under medium load.

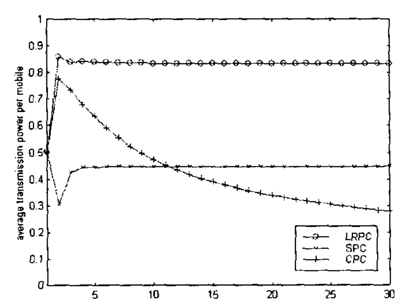

Fig. 9. Average transmission power per mobile under heavy load.