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Transmitter Power Control with Adaptive Safety Margins Based on Duration Outage

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Abstract. Transmitter power control in cellular networks is principally used to maintain a stable voice quality and to improve bandwidth utilization. There are two main types of quality based power control algorithms that have been extensively studied in the literature and implemented in practice. One is the *Signal to Interference Ratio (SIR) based* algorithms, and the other is the *Bit Error Rate (BER) based* algorithms.

Several practical issues however, were left open in the algorithms that have been proposed in the literature. The most critical issue in the SIR-based algorithms is how to determine the SIR target parameter; and the most critical one in the BER-based algorithms is how to derive good BER estimators in light of the rare occurrences of erroneous bits. Another issue that is shared by both algorithm types is the functional relation between the BER and the SIR. This relationship is required as one may serve as a control valve and the other as a control objective.

Determining the SIR target control parameter (in SIR or BER based algorithms) involves more than just a static transformation between the BER and the SIR. Since the actual SIR values is a stochastic process in nature, its variation and time correlation must be accounted for.

This paper addresses the problems described above by evaluating a *duration outage* probability of the underlying time process of the SIR values. We show that this probability can reasonably be approximated by a simple expression that relates between the *duration outage* probability and the SIR target control parameter. The distributed algorithm that is implied by this method is a *duration outage* based power control that uses an estimator for the most likely event (in contrast to BER-based algorithms).

Keywords: cellular networks, wireless, power control, outage probability, Gaussian processes, mobility.

1. Introduction and Problem Statement

Transmitter power control in cellular networks is principally used to maintain a stable voice quality and to improve bandwidth utilization. There are two main types of quality based power control algorithms that have been extensively studied in the literature and implemented in practice. One is the *Signal to Interference Ratio (SIR) based* algorithms, and the other is the *Bit Error Rate (BER) based* algorithms (see e.g., [1]). The following is a generic SIR-based Distributed Constrained Power Control (DCPC) that has been proposed in [4]. It has been shown there that if the link gains are stationary then both, the updated powers and the measured SIRs converge to unique values. In the linear power scale, DCPC updates the transmitter power by the simple iteration

$$p_i(t+dt) = \max\left\{P_{\min},\ \min\left\{P_{\max},\ \frac{\gamma}{\bar{\gamma}_i(t)}\, p_i(t)\right\}\right\}\ , \quad (1 \le i \le N)\,, \tag{1}$$

where N is the number of transmitters, $p_i(t)$ is the ith transmitter power at time t, $\bar{\gamma}_i(t)$ is the estimated SIR value at receiver i at time t, and $P_{\min}$ and $P_{\max}$ are minimum and maximum

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transmission powers. Finally but essentially, γ is a control parameter that sets the desirable convergence target of the process $\{\bar{\gamma}_i(t)\}$. The control parameter γ is selected in accordance with the ideal SIR value that is required by the decoder. From [4], mobiles that can be supported attain the SIR target γ, and those that cannot be supported, attain a lower SIR value.

The BER-based power control is closely related to the SIR-based one. Assume that there is a monotone relationship between the SIR and the BER functions. Then it has been shown in [7] that the following BER-based algorithm also converges. At discrete time instances t, the BER is estimated (rather than the SIR), and is mapped into its corresponding SIR. The SIR is then used to update the powers using the iteration in Equation (1). Note that since the BER estimator is time dependent, this BER-based algorithm implies a time dependent SIR target γ. A similar power control algorithm is used in CDMA IS-95. Also note that by using state-dependent values for $P_{\min}$ and $P_{\max}$, the powers can be updated in a manner that differences between successive power values will not exceed a given Δ.

These two types of power control algorithms left open several issues. The most critical issue in the SIR-based algorithm is how to determine the SIR target γ. Note that the SIR target is only an instrument to maintain a low BER which the decoder can cope with. From this exact reason, BER-based algorithms make more sense as the the estimated objective function at time t is used to control the objective function at time $t + dt$. Note however, that it may not necessarily imply a higher bandwidth utilization. More critical is the fact that erroneous bits are rare and therefore BER is hard to estimate. Had this problem been resolved, yet another one still exists. Namely, how to map the estimated BER value into the next transmission power. Usually, empirical functional relationships between the BER and SIR are used for that purpose. As these relationships are quite specific to system environments, it is not always clear how to use them in new circumstances.

Regardless of the algorithm type, determining the control target γ, that may be time-dependent, involves more than a static relationship between the BER and the SIR. Since the measured process $\{\gamma_i(t)\}$ is stochastic in nature, its variation and time correlation must be accounted for when setting the target control γ.

The impact of setting a static target γ on the actually achieved SIR has been studied in [2, 3, 12]. It has been shown there that the static SIR target must be set significantly higher than the desired one, otherwise an outage probability close to one is obtained. This phenomenon is explained by the fact that the actual SIR process is stochastic. It has been further shown that when the stochastic behavior is accounted for, the desired SIR can be maintained with high probability.

This paper addresses the issues described above by evaluating the *duration outage* probability (defined below) of the process $\{\gamma_i(t)\}$. In Section 2 we show that this probability can reasonably be approximated by a simple expression that relates between the *duration outage* probability and the control parameter γ. Using this relationship, the target control γ can be determined as a function of the required *duration outage* probability. In Section 3 we discuss the model assumption and propose an adaptation to the distributed power control algorithm given in Equation (1), that is based on this relation. The adapted algorithm sets a time-dependent SIR target as a function of the estimated *duration outage* probability. The estimation procedure is made efficient by estimating the rate of the most likely event where the process $\{\gamma_i(t)\}$ crosses its mean value, from which the probability of the *duration outage* rare event is readily obtained. Finally, in Section 4, we present numerical data that demonstrates how sensitive the SIR target is to the parameters affecting the $\{\gamma_i(t)\}$ process.

The main contribution of this study is by showing how to practically select a time dependent SIR target γ in one of the most studied power control schemes, (1). Compared with other algorithms the proposed one is more significant in the sense that it uses an efficient estimator, and also drives a more meaningful quality measure, the *duration outage* probability, to a desirable value. The innovative part is applying a more suitable quality measure (proposed in [9]), along with the observation that the correlation function of the log normal shadow fading has in fact a second derivative at the origin.

2. Duration Outage and Level Crossing

For better readability we hereinafter use the same notation to denote the SIR process of a generic transmitter i, $\{\gamma_i(t)\}$ in the dB scale, and omit the transmitter index. Let γ^0 be a predefined SIR constant under which the the signals can be decoded with a desired quality. That is, given that the process $\{\gamma_i(t)\}$ is constant in time and equals γ^0, then the service quality is satisfactory. For a given decoder, γ^0 can be determined in a laboratory.

The following definition is used in [9] to describe the SIR deterioration of the process $\{\gamma_i(t)\}$. *Let τ be a given time interval, define the duration outage event, E, as $\{\gamma_i(t)\}$ crossing below γ^0, and staying below that level for at least τ units of time.*

The *duration outage* better reflects the relation between SIR levels and BER, compared with the common definition of outage where $\tau = 0$. Indeed, bits are lost only if the SIR process stays below γ^0 for a minimum duration which disable recovery, or is noticeable. The objective of a power control algorithm is then to achieve

$$Pr(E) \leq \beta\,,$$

where $\beta > 0$ is an upper bound on the error probability.

In the rest of this paper we derive a distributed algorithm that achieves this objective. The algorithm facilitates classical results that have been obtained for rare level crossing events in Gaussian processes. The results are given in e.g., [8, 11], and were highlighted in [9] in the context of *duration outage*. The underlying assumptions that we make on the $\{\gamma(t)\}$ process are discussed in Section 3.

In this paper we confine ourselves to the DCPC algorithm defined in (1), and assume that the process $\{\gamma(t)\}$ is stationary and differential. For a given safety margin $\delta > 0$, consider the following reversed and shifted process version $\left\{X^\delta(t)\right\}$, given in the dB scale,

$$X^\delta(t) = (\gamma^0 + \delta) - \gamma(t)\,.$$

Let μ and σ^2 denote the stationary mean and variance of the process $\left\{X^\delta(t)\right\}$. Note that σ^2 is also the variance of $\{\gamma(t)\}$. Moreover, if the DCPC algorithm in (1) is applied with a SIR target of $(\gamma^0{+}\delta)$ and converges, then for supported mobiles we have $\mu = 0$. For non-supported mobiles we have $\mu > 0$.

A *duration outage* occurs if and only if the process $\left\{X^\delta(t)\right\}$ up-crosses level δ and stays above that level for at least τ units of time.

Since the process is differential we can define $\lambda = -\frac{d^2}{dt}R(t)|_{t=0}$, a parameter that determines the crossings rate of a stationary process (as shown below). $R(t)$ is the differential correlation function of $X^\delta(t)$ – see below. We discuss the existence of the derivative in Section 3.

It is well known that for stationary and differential processes with mean μ and variance σ^2, the rate of crossing level l, $r(l)$, is given by (see e.g., [11, p. 347]),

$$r(l) = \frac{\sqrt{\lambda}}{\pi \cdot \sigma} \exp\left\{-\frac{(l-\mu)^2}{2\sigma^2}\right\} .$$

The crossing rate of the mean value, μ, is then given by

$$r \equiv r(\mu) = \frac{\sqrt{\lambda}}{\pi \cdot \sigma} . \tag{2}$$

Crossing the mean value is the most frequent level crossing event, a property which we exploit below to derive an efficient estimator for λ.

To derive an explicit formula for $Pr(E)$ we further assume that $\{X^\delta(t)\}$ is a Gaussian process with a differential correlation function $R(t)$, and that it rarely crosses level δ. Let $E(\delta, \tau)$ be the *duration outage* probability associated with a safety margin δ and a duration outage τ. From the Gaussian and the rare crossing assumptions, it has been shown in [9] that

$$\begin{aligned} Pr(E(\delta,\tau)) \;\approx\; & \frac{\tau\sqrt{\lambda}}{2 \cdot \pi}\, e^{-\frac{\lambda}{8}\left(\tau^2+\frac{4}{\lambda}\right) g(\mu,\sigma,\delta)} \\ & + \sqrt{\frac{2}{\pi \cdot g(\mu,\sigma,\delta)}}\left(1-\Phi\left(\frac{\tau}{2}\sqrt{\lambda \cdot g(\mu,\sigma,\delta)}\right)\right) e^{-\frac{1}{2} g(\mu,\sigma,\delta)} , \end{aligned} \tag{3}$$

where $g(\mu, \sigma, \delta) = (\frac{\delta-\mu}{\sigma})^2$, $\Phi(x)$ is the standard normal cumulative distribution function (cdf), and $a \approx b$ is used to denote an asymptotic equality when the crossing level tends to infinity. Since $Pr(E(\delta, \tau))$ is decreasing in δ, we have the following.

CONCLUSION. *Given a duration outage interval τ, and the parameters λ and σ of the process $\{\gamma(t)\}$, then the smallest safety margin δ that corresponds to a given duration outage probability β, is resolved from Equation (3).*

In the next section we discuss the validity of of our assumptions about the process $\{X^\delta(t)\}$ in a cellular network, and derive an efficient estimator for λ that will be used in the power control algorithm.

3. Application to Power Control in Cellular Networks

To apply the conclusion of Section 2 to a cellular network one has to verify that the assumptions made in Section 2 are reasonable for the underlying process $\{\gamma(t)\}$.

When the fading of the signal power is dominated by the log normal shadow fading with zero dB log mean and σ^2 dB log variance, and there is only one interferer, then $\{\gamma(t)\}$ is Gaussian. This is the case where multi-path are filtered out or are not applicable. On the other hand, if there are many interferers, as in CDMA, their sum in the power scale and in the dB scale are both asymptotically normal (see Marlow's classical result in [10]). Hence, $\{\gamma(t)\}$ is Gaussian in the limiting cases. For a Manhattan-like micro-cellular TDMA system, it has been also shown by simulation in [3, Figure 4 or 7.3], that the SIR values under DCPC is also normal. As discussed in [13, Section 3.1] it is common practice to approximate the sum of independent log normal variables by another log normal variable with appropriately chosen parameters; and in a cellular systems with mobile users the independence among interferers is a sensible assumption. Thus, $\{\gamma(t)\}$ can be assumed to be Gaussian.

The other crucial assumption is that the correlation function of the log normal shadow fading, $R(t)$, has a second derivative at the origin. A commonly used correlation function for the log normal shadow fading of a mobile transmitter is the one proposed in [6] based on field measurements. The correlation as a function of time , in the dB scale, is given by

$$\exp\left(-\frac{v}{c_d} \mid t \mid\right),$$

where v is the mobile velocity and c_d is a correlation distance environmental parameter. This function has been derived to match discrete measurement data. As can be seen from [6, Figure 1], this function diverges from typical measurements in the area close to the origin. This suggests that the model should be refined in this area. Another anomaly of this function is the fact that it has an indefinite second derivative in the origin, resulting in an unbounded number of crossing any arbitrary level. This is not entirely aligned with data observed in practical systems. Although the model follows the measurements very closely when getting away from the origin, it seems not to predict well the correlation in the area close to the origin.

A nice refinement of this correlation function has been given in [5] by applying spectral smoothing to the exponential function from [6]. The function that has been proposed in [5] (defined below), perfectly follows the shape of the measured correlation function. Moreover, it does not diverge from it near the origin, as the function proposed in [6] does. The refined correlation function from [6] is given by

$$R(t) = \int_{-\infty}^{\infty} \exp\left(-\frac{v}{c_d} \mid x \mid\right) \frac{1}{0.99\sqrt{0.1 \cdot \pi}} \exp\left(-\frac{(t-x)^2}{0.1}\right) dx\,.$$

The 0.99 and 0.1 values in this function are particular parameters of the refined correlation function that are used to shape the curve around the origin. It is differential and its second derivative in the origin is given by

$$\lambda = \frac{4 \cdot \sigma^2}{0.99 \cdot 0.1^{3/2}\sqrt{\pi}} \int_0^{\infty} \left(1 - \frac{2x^2}{0.1}\right) \exp\left(-\left(\frac{v}{c_d} + \frac{x}{0.1}\right)x\right) dx\,.$$

This refined correlation function settles the differentiability assumption.

The third assumption is that crossing level δ is a rare event. The simulations made in [9, Figure 3] indicate that the asymptotic approximation for the *duration outage* probability agrees with the simulation results for duration intervals from *1.3–3 sec*, and over estimate for *0.5–1.3 sec*. Hence, using this approximation is safe as it over-estimates the probability.

Next we show how to use the *duration outage* probability $Pr(E(\delta, \tau))$ from Equation (3) to set the control parameter $\gamma = \gamma^0 + \delta$ in the power update algorithm (1).

Observe that to resolve δ from equation $Pr(E(\delta, \tau)) = \beta$, for a given τ and β, the values of σ and λ must be known. That is, the log variance and the crossings rate parameter of each transmitter need to be estimated. Hereinafter, we reinstate the transmitter index i in our notation.

The σ of the combined log normal is estimated by using the Fenton–Wilkinson approximation given in [13, Equation (3.7)]. Namely,

$$\sigma^2 = \sigma_0^2 + 2\ln(N)\,, \tag{4}$$

where σ_0^2 is the log variance of the shadow fading of a single transmitter, and $(N-1)$ are the number of interfering transmitters. (We assume that the background noise is negligible.) In the TDMA micro-cellular environment that has been simulated in [3], the number of dominating interferers were between 2–4.

A runtime efficient estimator for λ_i, $\hat{\lambda}_i$, is implied from Equation (2) by replacing the parameters with estimators. That is,

$$\hat{\lambda}_i = (\sigma \cdot \pi \cdot \hat{r}_i(\mu_i))^2 \,, \tag{5}$$

where $\hat{r}_i(\mu_i)$ is a runtime estimator of the rate that the process $\{\gamma_i(t)\}$ crosses its mean value μ_i. A straightforward estimator at time t, $\hat{r}_i(t, \mu_i)$, is given by

$$\hat{r}_i(t, \mu_i) = \frac{\text{Total no. of crossing } \hat{\mu}_i(t) \text{ during } [t-W, t]}{W} \,,$$

where W is a given time window, and

$$\hat{\mu}_i(t) = \text{Average value of } \gamma_i(t) \text{ during } [t-W, t] \,.$$

The time window W should be tuned to the operating environment. On one hand it should not be too large so it could be sensitive to the time dependency of the process $\{\gamma_i(t)\}$. On the other hand it should not be too small in order to leave room for sufficient crossings and therefore to a low estimator variance. The window size can be made adaptive to the estimated variance. Setting such window sizes is quite standard and the precise methods is beyond the scope of this paper.

Since crossing the mean is the most frequent crossing event, estimating λ_i is far more efficient than a direct estimation of BER.

AVERAGE SIR CROSSING POWER CONTROL: *Let* $\hat{\delta}_i(t)$ *be the safety margin obtained by solving Equation (3) using the* σ *from (4) and the estimator* $\hat{\lambda}_i$ *from (5). The Average SIR Crossing Power Control updates the power according to Equation (1) by using the time dependent dB SIR target* $(\hat{\delta}_i(t) + \gamma^0)$.

4. Safety Margin Sensitivity

In this section we show the sensitivity of the safety margin to the system parameters. In all the figures we represent the *duration outage* probability as a function of the safety margin in the dB scale. Figure 1 shows the sensitivity to changes in τ, Figure 2 shows the sensitivity to changes in λ, Figure 3 shows the sensitivity to changes in σ, and Figure 4 to changes in μ.

All figures demonstrate that the safety margin is sensitive to all these parameters. Thus, a careful setting of the outage duration interval is important. As to the other parameters. The sensitivity to λ and σ indicates that adaptive estimation is important. We use adaptive estimation only for λ, but one may wish to do so also for σ. E.g., by taking the sample variance of measurements from the process $\{\gamma_i(t)\}$.

The sensitivity to μ that is demonstrated in Figure 4 has no implication on the power control algorithm. It illustrates how the *duration outage* probability increases for mobiles that cannot attain the target $\gamma^0 + \delta$. For supported mobiles ($\mu = 0$), a safety margin of 3.25 dB is required to keep the *duration outage* probability below 0.05, given that $\sigma = 4$ dB, $\lambda = 4$ and $\tau = 3$ sec. With that safety margin, mobiles that have an average SIR that is 0.25 dB below

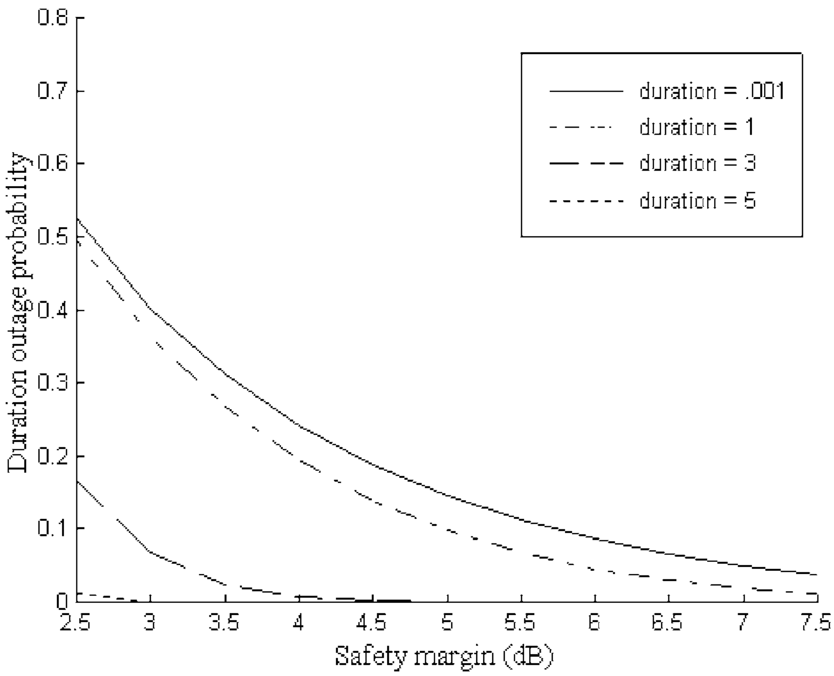

Figure 1. Safety margin sensitivity to the outage duration time τ with fixed parameters $\mu = 0$, $\lambda = 4$, $\sigma = 4$ dB and $\gamma^0 = 8$ dB.

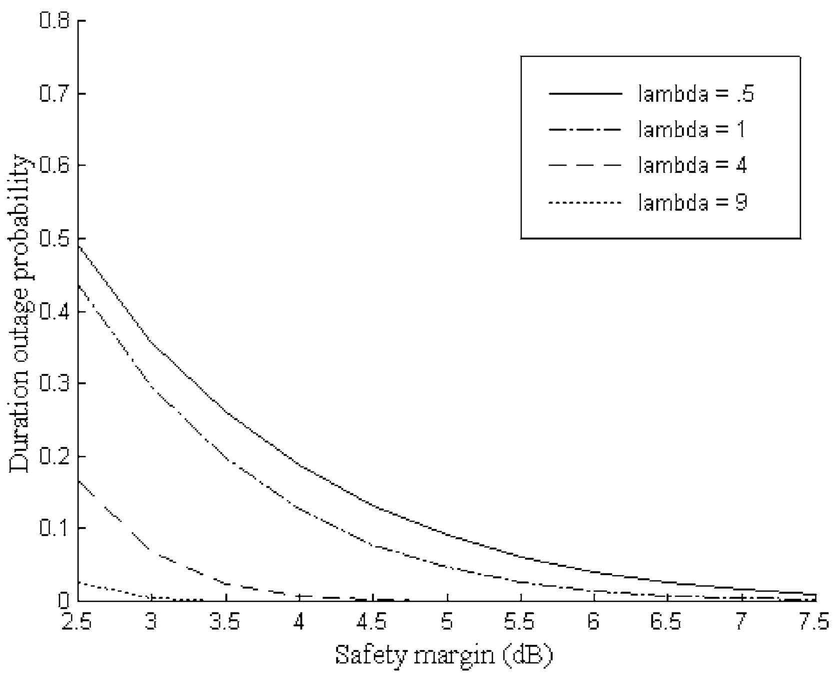

Figure 2. Safety margin sensitivity to the crossings rate λ with fixed parameters $\mu = 0$, $\tau = 3$ sec, $\sigma = 4$ dB and $\gamma^0 = 8$ dB.

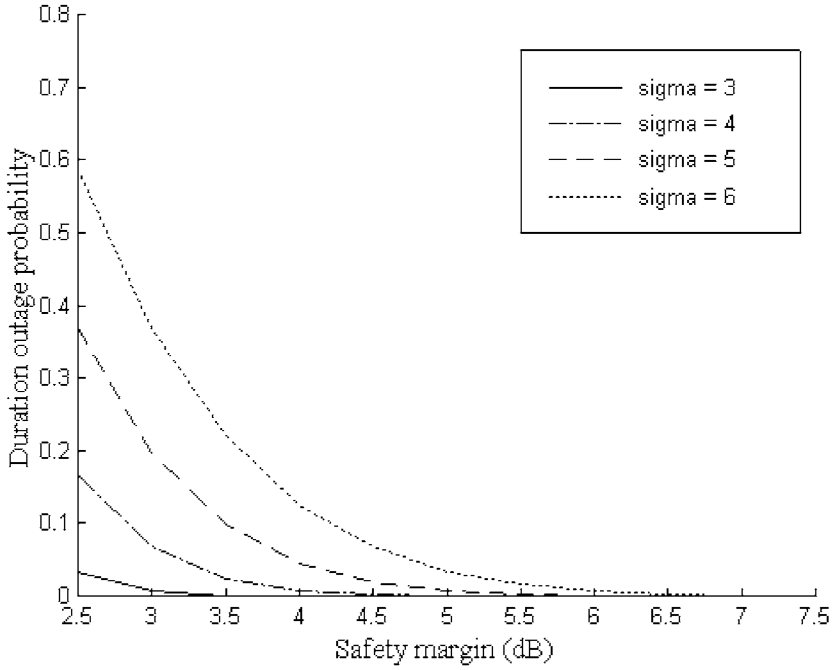

Figure 3. Safety margin sensitivity to the variance σ with fixed parameters $\mu = 0$, $\tau = 3$ sec, $\lambda = 4$ and $\gamma^0 = 8$ dB.

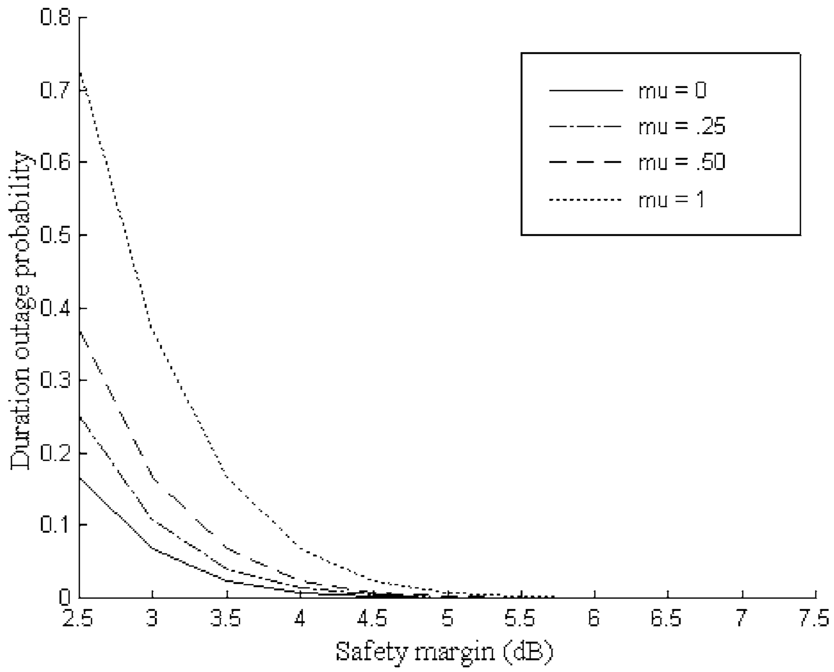

Figure 4. Safety margin sensitivity to the meam value μ with fixed parameters $\sigma = 4$ dB, $\tau = 3$ sec, $\lambda = 4$ and $\gamma^0 = 8$ dB.

the target have a *duration outage* probability of 0.09. Mobiles an average SIR that is 0.5 dB below the target have a *duration outage* probability of 0.15, and those that have an average SIR that is 1 dB below the target, have a *duration outage* probability of 0.28.

It worst noting that in these examples we have not studied the affect of γ^0. Its affect is manifested only through the values of μ that each mobile experience. The lower the γ^0 is, the more mobiles have $\mu = 0$.

5. Conclusions

In this study we addressed two basic issues in quality-based power control algorithms. One is how to determine a time dependent quality target in the power iteration equation, and the other is how to make the underlying estimation process of rare events efficient. We proposed a distributed algorithm that resolves these issues by relating the *duration outage* to the SIR level process and using classical results for rare level crossing events. We also discussed and argued that the model assumptions are valid in several cellular environments and demonstrated the sensitivity of the quality target to the environment parameters.

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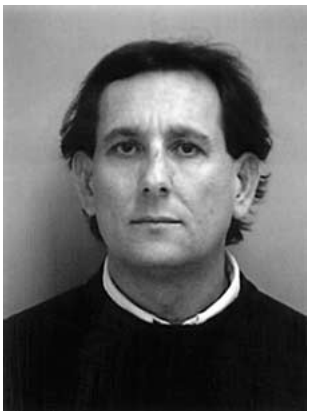

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