Asymptotically Optimal Transmission Power and Rate for CDMA Channels with MF and MMSE Receivers

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Abstract

The asymptotically combined optimal transmission power and rate control policy is derived for a DS-CDMA time varying fading channel with multiple user classes, random spreading codes and a receiver using either a conventional matched filter (MF) or a minimum mean square error (MMSE) multiuser detector. For a general objective function, the optimal policies are given by closed form functions of a single Lagrangian multiplier. The optimal policies are demonstrated by an application, where the transmission power is adapted to the channel fade variations, and the transmission rates are adapted to the tier containing the mobile. The effect of the number of tiers on the optimal transmission rate are presented for MF and MMSE receivers in an environment with Lognormal and Rayleigh fading. It is shown that with an MMSE receiver, there is a substantial increase in the total transmission rate, whereas only a negligible increase exists with a MF receiver.

Index Terms

Transmission power control, Transmission rate control, outage probability, Adaptive rate transmission, CDMA, Random spreading codes, Fading channels, MMSE multiuser receiver, Matched filter receiver.

I. Introduction

Advanced wireless networks support concurrent communication services over the same spectrum bandwidth, each having its own service attributes. Two critical attributes are transmission and bit error rates. The transmission rate of a service could be flexible and may take any value within some feasible set. Moreover, if there is no hard delay constraint on the service, the transmission rate could also vary in time. Since different transmission rates convey different merits, which may depend on the service, a rate utility function is associated with each service. A service is generally defined by its service attributes. For instance, if the attributes are transmission rate, bit error rate, utility function and channel gain, then all users having the same attribute values at a given moment comprise a service class (*user class*). The flexibility of allocating different transmission rates to different users at different times give rise to the *transmission rate control problem*.

Transmission rate over a communication channel is tightly coupled with the signal to interface ratio (SIR) at the receiver, and therefore with transmission power. Thus, transmission power and transmission rate control are naturally combined into one joint control problem.

The original objective of transmission power control [1]–[17] in Direct Sequence Code Division Multiple Access (DS-CDMA) cellular networks was to resolve the ‘near-far’ problem. Currently, it is also used to allocate spectrum bandwidth between users and to address problems rising from user mobility. Fast power control is particularly important when channel gain variations is fast, .e.g., as in Rayleigh fading channels.

Centralized power control algorithms, which maximize the minimum SIR or attain a common SIR target for all users, have been derived in [1]-[3] for a snapshot system, where all channel gains are fixed and known. Distributed power control algorithms, using the same snapshot model, have been studied in [4]–[12], where iterative algorithms converging to a common SIR target have been derived. In [4]–[12], except for [9], the convergence has been derived by assuming continuous power levels. Convergence properties with discrete power levels have been studied in [9]. In [12], an iterative power control algorithm with slow power update rate has been derived for combating fast fading, subject to an outage probability constraint. The underlying idea of the algorithm is to account for the fast variation via its distribution function.

Centralized and iterative power control algorithms may not adapt sufficiently fast to channel gain variations. Faster power adaptation exploiting the time correlation in the channel gain process have been studied in [10] [11]. Another power update acceleration method is used by algorithms based on channel gain estimation. An asymptotically optimal policy of this nature has been derived in [13] for a DS-CDMA channel with a linear minimum mean square error (MMSE) multiuser detector. Information theoretic power control algorithms [14]–[17] are based on channel gain side information (CSI) and assume optimal coding and detection method, i.e., having Shannon capacity. In [14], it has been shown that Shannon capacity of a single user fading channel, subject to a power budget constraint, is maximized by a ‘water-filling’ policy in time. This result has been extended in [17] for a DS-CDMA fading channel with an arbitrarily large number of users and processing gain. The minimum average power policy, which achieves the Shannon capacity region for multiple users, has been derived in [15]–[16].

Unlike transmission power control for DS-CDMA channels that requires fast adaptation, transmission rate control could also be useful at slow adaptation rates. Generally speaking, transmission rates are adapted to variations in the user classes. The variation rate depends on the class definition. If a user class represents a communication service such as voice, web-browsing or streaming, then class variations occur at user arrival and departure instances. If a user class represents the set of users at a given tier enclosed between two radii around the base station, then class variations occur at mobile transitions between tiers. These two class types imply transmission rate control at a relatively slower rate compared with power control. System models of this nature, referred to as a *slow-rate-fast-power control* systems, have been studied in [18]–[20] for a DS-CDMA channel with a conventional matched filter (MF) and with either random spreading codes or a ‘single implied spreading code’. By a single implied spreading code, it is meant that multiple access interference (MAI) is accounted as if all channels are cochannels.

System where the instantaneous transmission rate is adapted to channel gain variations as fast as the transmission power, referred to as *fast-rate-fast-power control* systems, have been studied in [21]–[23] for a DS-CDMA channel with a conventional MF and random spreading codes. Note that if the instantaneous transmission powers and rates are both functions of the instantaneous channel gains and there is no a priori classification into user classes, then the combined power and rate control collapses to the classical power control problem [17].

Transmission rate control algorithms and their performance depend on the multiple access method, the coding and the detection method [17], [24], [25], [28]. For instance, the DS-CDMA channel capacity with a conventional MF detector is lower than that of a channel with a linear MMSE multiuser detector [24]. The studies [18]-[23] have considered rate control problems for a channel with a conventional MF detector. This paper explores a DS-CDMA channel with both receiver types, the linear MMSE multiuser detector and the conventional MF receiver, and demonstrates a clear and significant difference between the two.

Slow-rate-fast-power controls have been considered in [18]–[20]. The performance of two decentralized slow-rate-fast-power algorithms have been studied in [18] for a DS-CDMA channel with random spreading codes and discrete rates. The algorithms aim at maximizing the total user utility in an interference-limited channel where transmission power is constrained by a peak power level. One algorithm facilitates a sub-gradient search in the corresponding Lagrangian function and the other is heuristic. Both algorithms use the iterative minimum power budget power control [7] to attain the respective SIR targets during each rate update step. A 'single implied spreading code' has been assumed in [19] [20] to model a downlink CDMA channel for data transmission. The control objective in both studies aim at maximizing the total user utility. The constraints, the power controls and the bit error models, however, are different. Assuming an underlying fast power control and some heuristic approximations in [19], the problem of rate adaptation at data segment boundaries lends itself to a constrained optimization problem. Then, the problem is solved by an iterative distributed algorithm with guaranteed convergence. In [20], both power and rate, are adapted at data segment boundaries, and a sub-optimal solution to the resulting constrained optimization program is advised. The sub-optimal algorithm there, also uses a sub-gradient search in the corresponding Lagrangian function.

Fast-rate-fast-power controls for a DS-CDMA channel with a conventional MF and random spreading codes [26] are considered in [21]–[23]. The power budget of two heuristic fast-rate-fast-power algorithms with CSI have been analyzed in [21]. Given a peak transmission power and a channel gain threshold, both algorithms limit the transmission power when the channel gain is below the threshold. For every channel gain, they either transmit at fixed rate and adapt the power, or vise versa. In [22], a centralized optimization problem has been defined for any given snapshot of all channel gains in the system. By approximating the bit error probability for large values of bit energy-to-interference ratio, the problem of minimizing the sum of powers subject to each user minimum effective transmission rate and to a maximum global transmission rate threshold, lends itself to a geometric program. The problem is then solved by a generic bisection search algorithm. In [23], it has been shown that the total transmission rate (including erroneous bits) subject to user-dependent peak transmission power constraints is maximized by a 'threshold bang-bang' policy. Namely, in the case of unlimited continuous user rates, each user transmits at peak transmission power and rate, if its peak transmission rate is above a threshold value. Otherwise, it does not transmit at all. In the case of continuous limited rates, at most one user at the threshold level may transmit at a fractional peak rate. The discrete rate case has been also considered, and the optimal solution requires a search for a threshold in an ordered list of size n, where n is the number of all feasible user rate combinations. The threshold value and the peak transmission rates are reevaluated whenever the channel gains change.

This paper presents a rigorous derivation of the asymptotically optimal slow-rate-fast-power control that maximizes the total user utility subject to a power budget constraint and to a peak transmission power level. The channels under consideration are DS-CDMA channels with random spreading codes and with either a linear MMSE multiuser detector or a conventional MF receiver. Asymptotically optimal is defined as in [17], referring to a channel with an arbitrarily large number of users and processing gain [13], [25], [28]. Asymptotic expressions are used due to analysis intractability of finite DS-CDMA systems. While providing excellent approximation for systems with practical number of users and values of the processing gain, the asymptotic facilitates an analytical closed-form solution, which is also simple to implement. The key features of the optimal policies for both, the MF and the MMSE receivers are:

- The optimal power and rate controls define two decoupled optimization problems.
- For every feasible rate vector, the optimal power control for each user is a *threshold cut-off policy* that inverts the controlled channel gain, if the gain is above a given threshold. Otherwise, it cuts off the transmission. The channel gain inversion is done so as to attain the optimal SIR targets determined by the optimal required rates. The cut-off threshold is pre-computed from the channel gain distribution.
- The optimal rates are given by a closed-form function of a single Lagrange multiplier. The optimal multiplier is determined by J independent fast bisection searches for a zero, where each search is done in a single-valued function and J is the number of user classes.
- For a channel with an MF receiver and a total rate utility function, the optimal policy is completely specified in a closed form.

The remainder of the paper is organized as follows. The system model and the problem are defined in Section II. The asymptotic optimal controls for a channel with an MMSE receiver and with an MF receiver are derived Section III and Section IV, respectively. The implementation aspects are discussed in Section V, and in Section VI, the optimal control is demonstrated by a numerical example of an application where a mobile class is determined by its distance from the base station. Finally, conclusions are presented in Section

VII.

II. System Model and Problem Definition

An uplink and a downlink of a DS-CDMA cellular network with time varying fading channels and N chips per symbol (aka processing gain) are considered. The transmitted signals in the uplink are detected at the base station using a linear MMSE multiuser detector, and in the downlink channel, they are detected at the mobile using a conventional MF receiver. For both links, the base station controls the transmission powers and transmission rates of K users from J different classes.

Each user class j, comprises $K_j = \lfloor \alpha_j N \rfloor$ users, where α_j is fixed. Let $K = \sum_{j=1}^{J} K_j$ and $\alpha = \sum_{j=1}^{J} \alpha_j$. For better clarity, the following synchronous channel model is assumed in this paper, however, the results can be extended to asynchronous channels as in [13, Section III E].

Following the convention from [24], the baseband synchronous received signal during the n^{th} symbol time interval in the presence of time variant fading and AWGN is given by

$$\boldsymbol{y}(n) = \sum_{i=1}^{K} x_i(n)\boldsymbol{s}_i(n)\sqrt{p_i(n)h_i(n)} + \boldsymbol{w}(n) \;, \tag{1}$$

where i is a user label.

In (1), for every symbol n and user i, $x_i(n)$ is the symbol information with $E[x_i^2(n)] = 1$, $\boldsymbol{s}_i(n)$ is the unit energy spreading code vector of length N, $\sqrt{h_i(n)}$ is the random channel gain and $p_i(n)$ is the user transmission power. The vector $\boldsymbol{w}(n)$ of length N denotes the AWGN with zero mean and power spectral density σ^2.

The baseband model in (1) represents frequency flat fading over the frequencies occupied by a symbol power, which is the appropriate model when sampling rate is not sufficiently fast to resolve multiple paths. (When sampling rate is sufficiently fast to resolve multiple paths, each symbol path should is modelled with its respective fading.) As usual, it is assumed that the channel gain $\{\boldsymbol{h}(n),\ n \geq 0\}$ process is stationary and ergodic. Note however, that the process could be correlated in time.

In a random spreading code model, $\boldsymbol{s}_i = \frac{1}{\sqrt{N}}(s_i(1), s_i(2), \ldots, s_i(N))^T$ is a random sequence, where $\{s_i(j)\}$ are assumed to be independently identically distributed random variables with zero means, unit variances and bounded fourth moments. Such sequences accurately model pseudo-random number (PN) long sequences used for some CDMA channels in IS-95, CDMA-2000 and W-CDMA. The spreading codes are chosen independently of the channel gains and once chosen, they are known by each transmitter-receiver pair.

For the sake of the power and rate control formulation, it is assumed that the receiver knows the gain of each channel during every symbol time n. In practice, channel gain estimators are used instead of the true unknown values. As will become apparent, only the optimal powers [13] are functions of the instantaneous channel gains, whereas the optimal rates are not. In [13], it has been shown how to transform the optimal gain-based power control policy into an estimator-based policy, while maintaining the required outage probability and power budget.

The combined transmission power and rate control defined below, aims at optimal bandwidth management in a DS-CDMA cellular network. In the presence of multiple user classes, it accounts for the different attribute values of each class. Since spreading codes are known by each side of the communicating parties and a synchronous channel is assumed, the complete channel information for a power and rate control policy at time n is the K-tuple $\boldsymbol{h}(n) = (h_1(n), h_2(n), \dots, h_K(n))$ and the AWGN. For every n, $\boldsymbol{h}(n) = \boldsymbol{h} = (h_1, h_2, \dots, h_K)$ will be referred to as the (system) state at time n.

For every power control policy, let $p_j(\boldsymbol{h})$ and $\gamma_j(\boldsymbol{h})$ denote the stationary transmission power and the instantaneous received SIR at state $\boldsymbol{h}$, respectively, corresponding to a user from class j. Given a common probability measure for all channel gains, let $F_j(h_j)$ be the continuous stationary marginal cumulative distribution function (cdf) of the channel gain for a user from class j , $1 \leq j \leq J$. Finite first and second moments are assumed.

Every user class is associated with an outage probability, $\epsilon_j < 1$, and a utility function, $U_j(\gamma_j)$. The utility function is defined as the utility per chip of the reliably transmitted symbols, given the SIR target allocated to user class j is γ_j, $0 \leq \gamma_j < \infty$. For every j, $U_j(\gamma_j)$ is assumed to be positive, twice differentiable and an increasing concave function.

A natural utility is the user symbol transmission rate per chip. For various coding, modulation and detection schemes it is given by $U_j(\gamma_j) = b_j \log_2(1 + \gamma_j)$, $b_j > 0$.

Communication of users from class j are *supported at target level* γ_j in state $\boldsymbol{h}$, if and only if $\left\{\gamma_j(\boldsymbol{h}) \geq \gamma_j\right\}$. The term *outage* will be used to indicate a state $\boldsymbol{h}$ where $\left\{\gamma_j(\boldsymbol{h}) < \gamma_j\right\}$. Communication at a given target level γ_j is possible only during supported states. Given that the SIR target allocated to class j is γ_j, the utility of users from class j is given by $U_j(\gamma_j)$.

The higher the allocated SIR target is, the higher is the transmission rate and the required transmission power. Increasing the transmission power has conflicting effects: (i) the transmission rate and its respective utility increase; (ii) more transmission power is consumed from each user power budget; and (iii) larger MAI power is generated. A proper transmission power and rate control can be obtained by allocating optimal SIR targets, γ_j^*, $1 \leq j \leq J$, and optimally controlling the transmission powers so as to maximize the total user utility subject to a total power budget constraint. In a slow-rate-fast-power control problem, SIR target allocation is a function of the long-term input parameters $\{\gamma_j\}$, $\{U_j(\gamma)\}$, $\{F_j(h)\}$ and the total average power budget $\overline{P}$, as well as a function of the relatively short term parameters $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_J)$, reflecting the proportions of users in each class. That is, the optimal SIR targets, $\{\gamma_j^*\}$, may change whenever $\boldsymbol{\alpha}$ changes.

In practice, there is also a hard upper bound constraint, P_{max}, on the mobile peak transmission power (e.g., 1 Watt in DCS 1800 and PCS 1900, and 2 Watt in GSM). For some systems, SIR target and outage probabilities may also be regraded as hard constraints. These constraints imply the following set of admissible power control policies.

Given any allocated SIR targets $\{\gamma_j\}$, a stationary power control policy $p_j(\boldsymbol{h})$ is *admissible* for class j, if $p_j(\boldsymbol{h}) \leq P_{max}$ for every $\boldsymbol{h}$, and there is a set of states $\boldsymbol{h}$, $\boldsymbol{H}_j$, satisfying the following requirements:

SIR requirement: For every state $\boldsymbol{h} \in \boldsymbol{H}_j$, $\gamma_j(h) \geq \gamma_j$. Namely, $\boldsymbol{H}_j$ is the supported set of users from class j at SIR target level γ_j.

Outage probability: $P(\boldsymbol{H}_j) \geq 1 - \epsilon_j$, where $P(\boldsymbol{H}_j) = \int_{\boldsymbol{h} \in \boldsymbol{H}_j} dF^{(K)}(\boldsymbol{h})$ is the probability of not having an outage.

A power control policy is *admissible*, if it is admissible for every class j. For a given SIR target, the set $\boldsymbol{H}_j$ above is referred to as the *SIR target supported set*. Note that by the definition, an admissible policy must satisfy $p_j(\boldsymbol{h}) \leq P_{max}$, for every state $\boldsymbol{h}$.

User utilities are determined by the allocated SIR targets, $\{\gamma_j\}$, and their cost are given by their average transmission power, where the average is taken with respect to the channel gain stationary distribution. The transmission powers at every state $\boldsymbol{h}$ clearly depends on the transmission power policy $\{p_j(\boldsymbol{h})\}$.

The combined transmission power and rate control optimization problem addressed in this paper is the following.

$$\max_{\{\boldsymbol{p}(\boldsymbol{h}),\boldsymbol{\gamma}\}} \sum_{j=1}^{J} \alpha_j U_j(\gamma_j), \tag{2}$$

subject to:

$$E\left[\mathcal{I}\left\{\boldsymbol{h} \mid \gamma_j(\boldsymbol{h}) < \gamma_j\right\}\right] \leq \epsilon_j, \quad 1 \leq j \leq J, \tag{3}$$

$$\sum_{j=1}^{J} \alpha_j E\left[p_j(\boldsymbol{h})\right] \leq \overline{P}\ , \tag{4}$$

$$p_j(\boldsymbol{h}) \leq P_{max}, \quad \forall\, \boldsymbol{h}, \quad 1 \leq j \leq J, \tag{5}$$

where $\mathcal{I}\left\{\cdot\right\}$ is the set indicator function, $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_J)$ and $\boldsymbol{p}(\boldsymbol{h}) = (p_1(\boldsymbol{h}), p_2(\boldsymbol{h}), \ldots, p_J(\boldsymbol{h}))$.

The objective function (2) is the total user utility per chip, and the left hand side of constraint (4) is the total average transmission power per chip. Since the transmission rate increases with the transmission power, $\overline{P}$ in constraint (4) serves as a reference level.

For finite N and K, all users are tightly coupled and finding the optimal control policy is intractable. Therefore, as in [13] [17] [25], an arbitrarily large system is considered, where $K_j = \lfloor \alpha_j N \rfloor$, $1 \leq j \leq J$, and $N \rightarrow \infty$. As demonstrated in [28], for power control policies with constant received power (as in our optimal control policy) and $N = 128$, MAI power is only $1 - 2$ dB away from the asymptotic MAI power value.

The combined asymptotically optimal transmission power and rate policy for an uplink DS-CDMA channel with a linear MMSE multiuser detector is derived in the next section.

III. Optimal Control for an MMSE Multiuser Detector

The MMSE multiuser detector is more efficient than the conventional MF and has a feasible implementation for base station receivers. Thus, it presents an attractive option for the uplink channels.

A. Optimal control

By [13, Theorem 4], if the power control policies are restricted to the form of $p_j(\boldsymbol{h}) = p_j(h_j)$, and a feasible policy exists for given SIR targets $\boldsymbol{\gamma}$ and outage probabilities $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_J)$, then the power control policy that minimizes the power budget in each user class j, $E\left[p_j(h_j)\right]$, subject to constraints (3) and (5), is the following threshold cut-off policy:

$$p_j^*(h_j) = \frac{R_j^*(\boldsymbol{\gamma})}{h_j} \mathcal{I}\left\{h_j \,|h_j \geq h_j^*\right\} \;, \tag{6}$$

where,

$$R_j^*(\boldsymbol{\gamma}) = \frac{\gamma_j \; \sigma^2}{1 - \sum\limits_{i=1}^{J} \frac{\alpha_i (1-\epsilon_i)\gamma_i}{1+\gamma_i}} \tag{7}$$

and each h_j^* is determined by $\int_0^{h_j^*} dF_j(h_j) = \epsilon_j$, independently of $\boldsymbol{\gamma}$.

The result in [13] has been derived for $F_j(h_j) = F(h_j)$ but can be extended to class-dependent distributions. Moreover, similarly to the derivation of [17, Lemma 4.10], it can be shown that the optimal power control policy without the restriction to policies of the form of $p_j(\boldsymbol{h}) = p_j(h_j)$, is also the one given in (6)–(7).

Accounting for the error margin induced by the asymptotic analysis when the processing gain N is finite requires intensive simulation, which is a subject for a subsequent study. A simple but naive way to account for finite N is by upper bounding the MAI power. From [13], $R_j^*(\gamma)$ in (7) is replaced with a similar expression where the noise power σ^2 is substituted with $\sigma^2 + E$. Here, E is the asymptotic error margin derived by simulating the channel interference. That is, asymptotic error acts as additional AWGN noise.

B. Optimal rate and power control combined

From Subsection III-A, the minimum power budget power control policy is given by a simple closed-form function that can be plugged into constraint (4) yielding the following combined rate and power control optimization problem.

$$\max_{\{\boldsymbol{\gamma}\}} \sum_{j=1}^{J} \alpha_j U_j(\gamma_j), \tag{8}$$

subject to:

$$\frac{\sum\limits_{j=1}^{J} \alpha_j c_j \gamma_j}{1 - \sum\limits_{i=1}^{J} \frac{\alpha_i(1-\epsilon_i)\gamma_i}{1+\gamma_i}} \leq \overline{P}, \tag{9}$$

$$\sum_{j=1}^{J} \frac{\alpha_j(1-\epsilon_j)\gamma_j}{1+\gamma_j} < 1, \quad \text{and} \quad \gamma_j \geq 0, \quad \forall \; j, \tag{10}$$

where $c_j = \sigma^2 \int\limits_{h_j^*}^{\infty} \frac{1}{h_j} dF_j(h_j)$.

Define,

$$g_j(\gamma_j) = \alpha_j c_j \gamma_j + \frac{\overline{P}\alpha_j(1-\epsilon_j)\gamma_j}{1+\gamma_j}. \tag{11}$$

The primal optimization problem (8)–(10) is equivalent to the following constrained optimization problem given in the standard form.

$$\min_{\{\boldsymbol{\gamma}\}} \left\{ -\sum_{j=1}^{J} \alpha_j U_j(\gamma_j) \right\}, \tag{12}$$

subject to:

$$\sum_{j=1}^{J} g_j(\gamma_j) - \overline{P} \leq 0 \quad \text{and} \quad \gamma_j \geq 0, \;\; \forall \; j. \tag{13}$$

It can be verified that the constraint sets in both problems are the same.

Note that the primal problem is **not a convex program**. Indeed, from the first and second derivatives of $g_j(\gamma_j)$, it can be observed that for every j, $g_j(\gamma_j)$ is a monotonically increasing concave function. Thus, the constraint set of $\boldsymbol{\gamma}$ is not convex. Nevertheless, it will be shown that under Condition 1 defined below, the optimal solution is derived from the optimal solution to the dual optimization problem. Furthermore, since the objective function (12) is monotonically non-increasing in every γ_j, the optimal solution is attained when constraint (13) is met with equality.

For every $\lambda > 0$, let

$$L(\boldsymbol{\gamma}, \lambda) = \lambda \left(\sum_{j=1}^{J} g_j(\gamma_j) - \overline{P} \right) - \sum_{j=1}^{J} \alpha_j U_j(\gamma_j) \tag{14}$$

denote the Lagrange function,

$$L_j(\gamma_j, \lambda) = \lambda g_j(\gamma_j) - \alpha_j U_j(\gamma_j) \tag{15}$$

denote its j^{th} component and $\boldsymbol{\gamma}^*(\lambda)$ denote the SIR target vector that solves the dual problem, i.e.,

$$\boldsymbol{\gamma}^*(\lambda) = \arg \inf_{\boldsymbol{\gamma} \geq 0} L(\boldsymbol{\gamma}, \lambda). \tag{16}$$

Also, let $\boldsymbol{\gamma}^*$ be the optimal solution to the primal problem and λ^* be the optimal solution to the dual problem, i.e.,

$$\lambda^* = \arg \max_{\lambda \geq 0} \inf_{\boldsymbol{\gamma} \geq 0} L(\boldsymbol{\gamma}, \lambda).$$

Since the problem is not convex, a further condition is required to facilitate the derivation of efficient optimal solution as with convex programming. The next condition, assuring that no user is starving under the optimal solution, addresses this target.

Condition 1: There exists a positive Lagrange multiplier ('shadow price'), $\overline{\lambda}$, such that the optimal dual solution, $\boldsymbol{\gamma}^*(\overline{\lambda})$, satisfies constraint (13) and each of its components is strictly positive.

As can be seen from the proof of Proposition 1 below, Condition 1 forces a strictly positive optimal solution, which is equivalent to constraint (13) being active. The condition also has a simple and plausible interpretation.

Condition 1 provides a sufficient condition for expressing the optimal solution in closed-forms for some utility functions, or to guarantee the convergence of the search algorithm specified in Section III-C. Its pre-verification for an algorithmic solution is not required. If the algorithm converges, the optimal solution is found, otherwise it is not. The algorithm may also converge when Condition 1 does not hold. For a closed-form solution, pre-verification may be required. An example of a verification procedure is shown below for a CDMA MMSE channel, where the utility function is given by the rate of reliable transmitted symbols.

The next proposition lays the foundation for the optimal transmission rate control policy.

Proposition 1: Assume that Condition 1 holds true.

(a) If $\lambda_1 < \lambda_2 \leq \overline{\lambda}$, then $\gamma_j^*(\lambda_1) \geq \gamma_j^*(\lambda_2) > 0$, for all j.

(b) There exists an optimal primal-dual solution pair $(\boldsymbol{\gamma}^*, \lambda^*)$ that solves the primal and dual problems.

Proof: By the separable structure of the primal problem (12)–(13), the minimization in (16) can be decomposed into the following J independent minimization problems:

$$\gamma_j^*(\lambda) = \arg \inf_{\gamma_j \geq 0} \left(\lambda g_j(\gamma_j) - \alpha_j U_j(\gamma_j)\right), \;\; \forall \;\, j. \tag{17}$$

Let $\nabla L_j(\gamma_j, \lambda)$ and $\nabla^2 L_j(\gamma_j, \lambda)$ denote the first and the second derivatives of $L_j(\gamma_j, \lambda)$ with respect to γ_j, respectively.

Every positive local minimum of $L_j(\gamma_j, \lambda)$, $\tilde{\gamma}_j(\lambda)$, is a solution to $\nabla L_j(\gamma_j, \lambda) = 0$ that satisfies $\nabla^2 L_j(\gamma_j, \lambda) > 0$. That is, a zero-level up-crossing point of $\nabla L_j(\gamma_j, \lambda)$. From (11) and (15),

$$\nabla L_j(\gamma_j, \lambda) = \lambda \left(\alpha_j c_j + \frac{\overline{P}\alpha_j(1-\epsilon_j)}{(1+\gamma_j)^2}\right) - \alpha_j \nabla U_j(\gamma_j), \tag{18}$$

implying that $\nabla L_j(\gamma_j, \lambda)$ increases with λ for every given γ_j. Thus, from the zero-level up-crossing property above, each local minimum $\tilde{\gamma}_j(\lambda)$ decreases with λ.

Part (a) of the proposition follows from the continuity of $\nabla L_j(\gamma_j, \lambda)$ and the fact that $\gamma_j^*(\overline{\lambda}) > 0$.

To ascertain part (b), it will be first shown that there is a positive Lagrange multiplier $\underline{\lambda}$ such that

$$\sum_{j=1}^{J} g_j(\gamma_j^*(\underline{\lambda})) > \overline{P}. \tag{19}$$

Since $g_j(\gamma_j)$ is monotonically increasing to infinity, there is a $\boldsymbol{\gamma}^0$ such that

$$\sum_{j=1}^{J} g_j(\gamma_j^0) > \overline{P}. \tag{20}$$

Let

$$\lambda_j = \frac{\alpha_j \nabla U_j(\gamma_j^0)}{\alpha_j c_j + \overline{P}\alpha_j(1-\epsilon_j)}.$$

For every $\lambda \leq \lambda_j$, the concavity of $U_j(\gamma_j)$ and (18) imply that

$$\begin{aligned} \nabla L_j(\gamma_j, \lambda) \;\; &< \lambda(\alpha_j c_j + \overline{P}\alpha_j(1-\epsilon_j)) - \alpha_j \nabla U_j(\gamma_j^0) \\ &\leq 0, \quad \forall \; \gamma_j \leq \gamma_j^0. \end{aligned} \tag{21}$$

That is, $L_j(\gamma_j, \lambda)$ is decreasing for all $\gamma_j \leq \gamma_j^0$, implying that $\gamma_j^*(\lambda) \geq \gamma_j^0$. Fixing

$$\underline{\lambda} = \min_j \lambda_j, \tag{22}$$

the monotonicity of each $g_j(\gamma_j)$ and (20) assure that inequality (19) holds true.

Combining (19) with Condition 1, the continuity of $\nabla L_j(\gamma_j, \lambda)$ and $L_j(\gamma_j, \lambda)$ along with the fact that for every λ, $\underline{\lambda} \leq \lambda \leq \overline{\lambda}$, $\gamma_j^*(\lambda)$ is a positive local minimum, imply that there exists a $\lambda^* > 0$ such that

$$\sum_{j=1}^{J} g_j \left(\gamma_j^*(\lambda^*) \right) = \overline{P}. \tag{23}$$

To complete the proof, note that the pair $(\boldsymbol{\gamma}^*(\lambda^*), \lambda^*)$ satisfies:

- **(i)** **Primal feasibility:** See (23).
- **(ii)** **Dual feasibility:** $\lambda^* \geq 0$.
- **(iii)** **Lagrangian optimality:** $\boldsymbol{\gamma}^*(\lambda^*) = \arg\inf_{\boldsymbol{\gamma} \geq 0} L(\boldsymbol{\gamma}, \lambda^*)$.
- **(iv)** **Complementary slackness:** $\lambda^* \left(\sum_{j=1}^{J} g_j \left(\gamma_j^*(\lambda^*) \right) - \overline{P} \right) = 0$.

By [29, Proposition 5.1.4], the pair $(\boldsymbol{\gamma}^*(\lambda^*), \lambda^*)$ is the optimal primal-dual solution pair. ■

The following corollary, resulting from the fact that conditions (i)–(iv) in the proof of Proposition 1 are necessary and sufficient, is used to derive the optimal rate control.

Corollary 1: If a solution pair $(\boldsymbol{\gamma}^*, \lambda^*)$ satisfies conditions (i)–(iv), then $\boldsymbol{\gamma}^*$ is the optimal solution to the primal problem (12)–(13).

■

If the SIR achieved by each user from class j is γ_j, the rate of reliable transmitted symbols in the MMSE CDMA channel is $\frac{1}{2} \log_2(1 + \gamma_j)$ [25, Proposition VI.1], [28, Section II]. Therefore, taking

$$U_j(\gamma_j) = \frac{(1 - \epsilon_j)}{2} \log_2(1 + \gamma_j), \tag{24}$$

the optimization objective function (8) becomes the total number of symbols per chip that are reliably transmitted in the channel. For this utility function, the optimal SIR targets, $\boldsymbol{\gamma}^* \stackrel{def}{=} \boldsymbol{\gamma}^*(\lambda^*)$, are explicitly specified by the following theorem.

Theorem 1: If Condition 1 holds true and $U_j(\gamma_j) = b_j \log_2(1 + \gamma_j)$, $b_j > 0$, $1 \leq j \leq J$, then

$$\gamma_j^* = \frac{\frac{b_j \log_2(e)}{\lambda^*} + \sqrt{\left(\frac{b_j \log_2(e)}{\lambda^*} \right)^2 - 4c_j(1 - \epsilon_j)\overline{P}}}{2c_j} - 1. \tag{25}$$

Proof: By Proposition 1, each optimal γ_j^* is a strictly positive local minimum of $L_j(\gamma_j, \lambda^*)$, and therefore a solution to $\nabla L_j(\gamma_j, \lambda) = 0$ that satisfies $\nabla^2 L_j(\gamma_j, \lambda) > 0$.

It can be verified that if $U_j(\gamma_j) = b_j \log_2(1+\gamma_j)$, the equality $\nabla L_j(\gamma_j, \lambda^*) = 0$ implies

$$\gamma_j^* = \frac{\frac{b_j \log_2(e)}{\lambda^*} \pm \sqrt{\left(\frac{b_j \log_2(e)}{\lambda^*}\right)^2 - 4c_j(1-\epsilon_j)\overline{P}}}{2c_j} - 1$$

and the inequality $\nabla^2 L_j(\gamma_j, \lambda) > 0$ implies (25). ■

Note that the optimal $\boldsymbol{\gamma}^*$ depends on the number of users in each class, $\boldsymbol{\alpha}$, only through the optimal power control (6)–(7). Also, for every Lagrange multiplier λ, $\gamma_j^*(\lambda)$ is given by (25) after replacing λ^* with λ. Thus, $\gamma_j^*(\lambda)$ is determined from local parameters and only λ^* requires global knowledge.

Also note that if Condition 1 holds true, the optimal γ_j^* must be a solution to $\nabla L_j(\gamma_j, \lambda) = 0$. Thus, verification of Condition 1 is essential to know when Theorem 1 applies.

The verification is as follows. In (25), replace λ^* with λ and define

$$\hat{\lambda}_j = \frac{b_j \log_2(e)}{\sqrt{4c_j(1-\epsilon_j)\overline{P}}} \quad \text{and} \quad \hat{\lambda} = \max_j \hat{\lambda}_j.$$

By (25), $\hat{\lambda}$ is the maximum Lagrange multiplier for which all $\{\gamma_j^*(\lambda)\}$ are real-valued. Next, compute

$$\overline{\lambda} = \max\left\{\lambda \leq \hat{\lambda} : \gamma_j^*(\lambda) > 0, \ \ \forall\, j\right\}. \tag{26}$$

Since each $\gamma_j^*(\lambda)$ decreases with λ and each $g_j(\gamma_j)$ increases with γ_j, Condition 1 holds true if and only if $\boldsymbol{\gamma}^*(\overline{\lambda})$ is feasible for $\overline{\lambda}$ defined in (26).

Closed-form solutions for other 'nice' utility functions can be derived similarly to the derivation of Theorem 1. Essentially, for such cases, $\nabla L_j(\gamma_j, \lambda) = 0$ has a closed-form unique solution given as a function of λ. Exploring its structure as done above, could lead to the verification of Condition 1.

An algorithm that finds the optimal SIR targets for a general utility function is defined in the next subsection.

C. Algorithm

Computing $\boldsymbol{\gamma}^*$ comprises two loops. The inner loop solves the dual problem, i.e., computes $\boldsymbol{\gamma}^*(\lambda)$, for every $\lambda > 0$ with a positive $\boldsymbol{\gamma}^*(\lambda)$. The outer loop updates λ and checks for convergence.

The inner loop of the algorithm is as follows.

Function: $\boldsymbol{\gamma}^*(\lambda) = SolveDualProblem(\lambda, \underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}})$

For every j, $1 \leq j \leq J$, do:

1. Find all the roots of $\nabla L_j(\gamma_j, \lambda)$ in the interval $[\underline{\gamma}_j, \overline{\gamma}_j]$ for which $\nabla^2 L_j(\gamma_j, \lambda) > 0$;

2. Set $\gamma_j^*(\lambda) = \arg\min_m \left\{ L_j \left(\gamma_j^m(\lambda), \lambda \right) \right\}$, where $\{(\gamma_j^m(\lambda)\}$ are the roots found in step 1;

End for.

Return $\boldsymbol{\gamma}^*(\lambda)$.

If the number of roots of $\nabla L_j(\gamma_j, \lambda)$ in the interval $[\underline{\gamma}_j, \overline{\gamma}_j]$ is small (which is usually the case), finding them is quite efficient and very often are given in a closed-form formula. As shown in Theorem 1, when the utility function is $U_j(\gamma_j) = b_j \ln(1+\gamma_j)$, there is only one root in the relevant interval $[\underline{\gamma}_j, \overline{\gamma}_j]$. A general condition ensuring a unique root in the relevant interval is the following.

Condition 2: For every class j and constant $a_j > 0$, there is a $\gamma_j(a_j)$, such that

$$\nabla^2 U_j(\gamma_j) \begin{cases} \geq -\frac{a_j}{(1+\gamma_j)^3}, & \text{for } \gamma_j \leq \gamma_j(a_j), \\ \leq -\frac{a_j}{(1+\gamma_j)^3}, & \text{for } \gamma_j \geq \gamma_j(a_j). \end{cases} \tag{27}$$

The geometrical interpretation of (27) is as follows. For every Lagrange multiplier $\lambda > 0$, let $a_j^\lambda = 2\lambda \overline{P}(1-\epsilon_j)$. The point $\gamma_j(a_j^\lambda)$ is the inflection point of the curve $L_j(\gamma_j, \lambda)$. That is, the point where the curvature changes from concavity to convexity. Thus, $\nabla L_j(\gamma_j, \lambda)$ has at most three roots among which only the largest (if exists) is a local positive minimum. Combined with Condition 1, a local minimum always exists in the relevant interval. Moreover, for every $\lambda > 0$, it follows from (18) and (27) that $\nabla L_j(\gamma_j, \lambda)$ decreases for $0 \leq \gamma_j \leq \gamma_j(a_j^\lambda)$ and increases for $\gamma_j \geq \gamma_j(a_j^\lambda)$.

In the outer loop, the algorithm searches the optimal λ^* that satisfies constraint (23). Since $g_j(\gamma_j)$ is monotonically increasing and continuous for every j, part (a) of Proposition 1 imply that a binary search in the interval $\left[\underline{\lambda}, \overline{\lambda}\right]$ converges to the optimal λ^*, where $\overline{\lambda}$ and $\underline{\lambda}$ are determined by Condition 1 and (22), respectively.

The outer loop of the algorithm is as follows.

Function: $\boldsymbol{\gamma}^* = SolvePrimalProblem$

1. Initialize:

1.1 Fix the error tolerance $\eta > 0$ and initialize the search interval $[\underline{\lambda}, \overline{\lambda}]$ with the values determined by Condition 1 and (22).

1.2 Compute $\underline{\boldsymbol{\gamma}} = SolveDualProblem(\overline{\lambda}, \boldsymbol{\delta}, \infty)$, where $\boldsymbol{\delta} > 0$ is an arbitrary small vector.

1.3 Compute $\overline{\boldsymbol{\gamma}} = SolveDualProblem(\underline{\lambda}, \underline{\boldsymbol{\gamma}}, \infty)$.

1.4 Set $\lambda = (\underline{\lambda} + \overline{\lambda})/2$ and compute $\boldsymbol{\gamma} = SolveDualProblem(\lambda, \underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}})$.

2. Loop:

While $\left(\left|\sum_{j=1}^{J} g_j\left(\gamma_j\right) - \overline{P}\right| > \eta\right)$ **do:**

If $\left(\sum_{j=1}^{J} g_j\left(\gamma_j\right) \leq \overline{P} - \eta\right)$, set $\overline{\lambda} = \lambda$ and $\underline{\boldsymbol{\gamma}} = \boldsymbol{\gamma}$;

Else, set $\underline{\lambda} = \lambda$ and $\overline{\boldsymbol{\gamma}} = \boldsymbol{\gamma}$.

Update: Set $\lambda = (\underline{\lambda} + \overline{\lambda})/2$ and Compute $\boldsymbol{\gamma} = SolveDualProblem(\lambda, \underline{\boldsymbol{\gamma}}, \overline{\boldsymbol{\gamma}})$.

End while

3. Return $\boldsymbol{\gamma}$.

The *SolvePrimalProblem* algorithm is iteratively closing the duality gap and its convergence is assured by Proposition 1. Here, the verification of Condition 1 is not required. If the duality gap is closed, the result is optimal; otherwise, it is not. The duality gap can be closed even when Condition 1 does not hold, since it is not a necessary condition.

The combined asymptotically optimal transmission power and rate policy for a downlink DS-CDMA channel with a conventional MF receiver is derived in the next section.

IV. Optimal Control for a Conventional MF Detector

Multiuser detection at the mobile side is not yet practical with current technology. Therefore, a conventional matched filter receiver is assumed for the downlink channel.

A straightforward corollary of [28, Proposition 3.3] is the following. Let $p_j(h_j)$ be a stationary power control policy used by the base station for transmitting to a user from class j, and $1/\beta^*(\boldsymbol{p})$ be the combined asymptotic interference power introduced by the AWGN and the transmissions to all other MAI users.

If all the receivers use the conventional MF, then $\beta^*(\boldsymbol{p})$ is given by

$$\beta^*(\boldsymbol{p}) = \frac{1}{\sigma^2 + \sum_{j=1}^{J} \alpha_j E_{h_j}\left[h_j p_j(h_j)\right]}. \tag{28}$$

Similar to the derivation in [13], it can be shown that if a feasible policy exists for given SIR targets $\boldsymbol{\gamma}$ and outage probabilities $\boldsymbol{\epsilon}$, the power control policy which minimizes the power budget of each class j, $E_{h_j}\left[p_j(h_j)\right]$, subject to the constraints in (3) and (5) is the following threshold cut-off policy:

$$p_j^*(h_j) = \frac{R_j^*(\boldsymbol{\gamma})}{h_j} \mathcal{I}\left\{h_j \left| h_j \geq h_j^*\right.\right\}\ , \tag{29}$$

where

$$R_j^*(\boldsymbol{\gamma}) = \frac{\gamma_j\ \sigma^2}{1 - \sum_{i=1}^{J} \alpha_i (1-\epsilon_i)\gamma_i}, \tag{30}$$

and h_j^* is determined by $\int_0^{h_j^*} dF_j(h_j) = \epsilon_j$, independently of $\boldsymbol{\gamma}$.

As for the uplink channel case in Subsection III-B, the minimum power budget power control policy is given by a simple closed form function that can be plugged into constraint (4) yielding the following combined rate and power control optimization problem.

$$\max_{\{\boldsymbol{\gamma}\}} \sum_{j=1}^{J} \alpha_j U_j(\gamma_j), \tag{31}$$

subject to:

$$\frac{\sum\limits_{j=1}^{J} \alpha_j c_j \gamma_j}{1 - \sum\limits_{i=1}^{J} \alpha_i (1-\epsilon_i)\gamma_i} \leq \overline{P}, \tag{32}$$

$$\sum_{j=1}^{J} \alpha_j (1-\epsilon_j)\gamma_j < 1, \quad \text{and} \quad \gamma_j \geq 0, \quad \forall \ j, \tag{33}$$

where $c_j = \sigma^2 \int\limits_{h_j^*}^{\infty} \frac{1}{h_j} dF_j(h_j)$.

Similarly to the derivation in Section III-A, the optimization program (31)–(33) is equivalent to the following convex constrained program with linear constraints.

$$\min_{\{\boldsymbol{\gamma}\}} \left\{ -\sum_{j=1}^{J} \alpha_j U_j(\gamma_j) \right\}, \tag{34}$$

subject to:

$$\sum_{j=1}^{J} \alpha_j \left(c_j + \overline{P}(1-\epsilon_j) \right) \gamma_j - \overline{P} \leq 0. \tag{35}$$

Note that the objective function (34) is monotonically non-increasing in every γ_j and the constraint function in (32) is linearly increasing. Thus, an optimal solution is obtained when constraint (33) is met with equality.

For every Lagrange multiplier $\lambda > 0$, let $\gamma_j^*(\lambda)$ be the unique solution to

$$\lambda \left(c_j + \overline{P}(1-\epsilon_j) \right) = \nabla U_j(\gamma_j),$$

if one exists; and zero otherwise.

Since for every j, $\gamma_j^*(\lambda)$ continuously decreases with λ, there is a λ^* for which constraint (33) is met with equality for $\gamma_j^*(\lambda^*)$. The λ^* can be found by a bisection search. Standard application of Karush-Kuhn-Tucker

(KKT) optimality conditions to separable constrained convex programs (alternatively, conditions (i)–(iv) in the proof of Proposition 1) reveals that $\{\gamma_j^*(\lambda^*)\}$ are the optimal SIR targets. A specific closed form solution is given below for the following specific utility function.

If the SIR achieved by each user from class j is γ_j, the rate of reliable transmitted symbols in the MF CDMA channel is $\frac{1}{2}\log_2(1+\gamma_j)$ [25, Proposition IV.1]. Therefore, taking $U_j(\gamma_j)$ as in (24), the optimization objective function (31) becomes the total number of symbols per chip, which are reliably transmitted in our MF CDMA channel. For this utility function, $\boldsymbol{\gamma}^*$ takes on the following specific closed form solution.

Let $\pi = (\pi_1, \pi_2, \ldots, \pi_J)$ be the permutation of the class labels ordered in an increasing order of $\frac{b_j \log_2(e)}{c_j+(1-\epsilon_j)\overline{P}}$. Define,

$$\lambda^n = \frac{\log_2(e)\sum_{j=1}^{n}\alpha_{\pi_j}b_{\pi_j}}{\overline{P}+\sum_{j=1}^{n}\alpha_{\pi_j}\left(c_{\pi_j}+(1-\epsilon_{\pi_j})\overline{P}\right)}, \quad 1 \le n \le J.$$

Simple algebra shows that for every $n, 1 \le n \le J-1$,

$$\lambda^n < \lambda^{n+1} \qquad \text{if and only if} \qquad \lambda^n < \frac{\log_2(e) b_{\pi_{n+1}}}{c_{\pi_{n+1}}+(1-\epsilon_{\pi_{n+1}})\overline{P}}.$$

Thus, we may define

$$\lambda^* \stackrel{def}{=} \lambda^{n^*} = \max\left\{\lambda^n \middle|\, \lambda^n < \frac{\log_2(e) b_{\pi_{n+1}}}{c_{\pi_{n+1}}+(1-\epsilon_{\pi_{n+1}})\overline{P}}\,, 1 \le n \le J-1\right\}. \tag{36}$$

With these definitions, the optimal SIR targets are readily available by following theorem.

Theorem 2: If $U_j(\gamma_j) = b_j\log_2(1+\gamma_j)$, $b_j > 0$, $1 \le j \le J$, then

$$\gamma_j^* = \left[\frac{b_j\log_2(e)}{\lambda^*\left(c_j+(1-\epsilon_j)\overline{P}\right)} - 1\right]^+, \tag{37}$$

where λ^* is given by (36) and $[x]^+ = x$ if x is positive, and 0 otherwise.

Proof: By noting that $\gamma_{\pi_n}^* > 0$ if and only if $n \le n^*+1$, it is straightforward to verify that conditions (i)–(iv) in the proof of Proposition 1 hold true. Alternatively, that KKT optimality conditions are satisfied. ■

Some of the implementation aspects are discussed in the next section.

V. Implementation aspects

For most common utility functions, the optimal transmission powers and rates can explicitly be derived as in Theorems 1 and 2. For such cases, given the channel fade marginal distributions $\{F_j(h_j)\}$ and the other system fixed parameters, the optimal SIR targets are pre-computed in the outset and do not require adaptation to the instantaneous channel gains. Since these system parameters change relatively slow, the optimal SIR targets are easily updated.

For the downlink channels, which use MF receivers, the optimal SIR targets are given in a closed form and are evaluated at the base station, where needed. For the uplink channels, which use MMSE receivers, the optimal SIR targets are also evaluated at the base station using the algorithm from Section III-C that finds the optimal Lagrange multiplier. Depending on the power control implementation (see below), the optimal SIR target can be sent to each mobile, if required.

Given the channel gain marginal distributions $\{F_j(h_j)\}$ and the optimal SIR targets, the optimal transmission power assigned to each user from class j requires only the knowledge of its instantaneous channel gain, h_j, which in practice, is replaced by an estimator. It has been shown in [13], how to transform the optimal power control policy based on h into a policy based on an estimator $\hat{h}$, while maintaining a desirable outage probability and power budget. Since in CDMA, the uplink and downlink channels are not reciprocal, the uplink channel is best estimated at the receiver and the required power update is sent to each mobile. With such implementation of an uplink power control, the optimal SIR targets computed at the base station are not sent to the mobiles.

VI. Application

Adapting the transmission rate to the channel condition may improve bandwidth efficiency in cellular networks. A simple and powerful scheme is the following quasi-static adaptation based on the distance between the mobile and the base station. The cell is partitioned into J tiers determined by J radii. At any moment, all mobiles located in tier j form the user class j, $1 \leq j \leq J$. The expected gain in a channel used by a mobile from class j is given by its average exponential path loss, which depends on the tier radii. Thus, the distribution functions, $\{F_j(h)\}$, are all different.

It is expected that the total optimal transmission rate will increase with the numbers of tiers. Moreover, the closer the tier is to the base station, the larger its optimal allocated rate is expected to be. Such quasi-static transmission rate adaptation is one application of our optimal transmission power and rate control problem. The implementation requires only rough estimation of the distance between the controlled mobile and the base station. Since mobile distribution over the tiers, as reflected by $\boldsymbol{\alpha}$, varies relatively slow, it does not present a computational burden on the implementation.

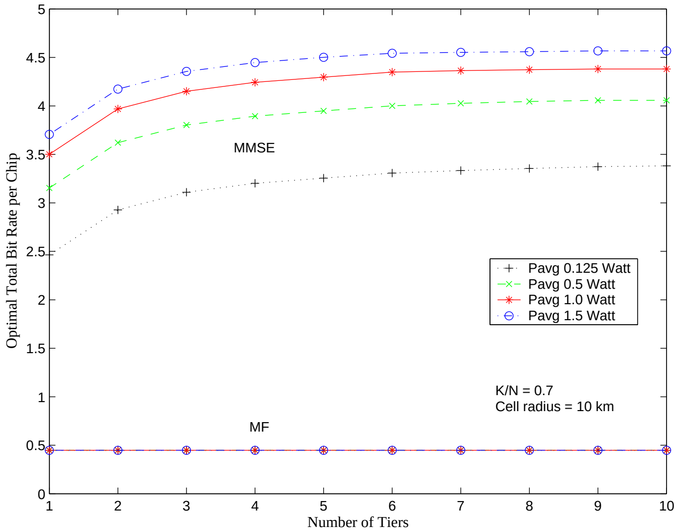

Fig. 1. Optimal total bit rate vs. the number of tiers for multiple power budget values with $\alpha = 0.7$ and cell radius 10 km.

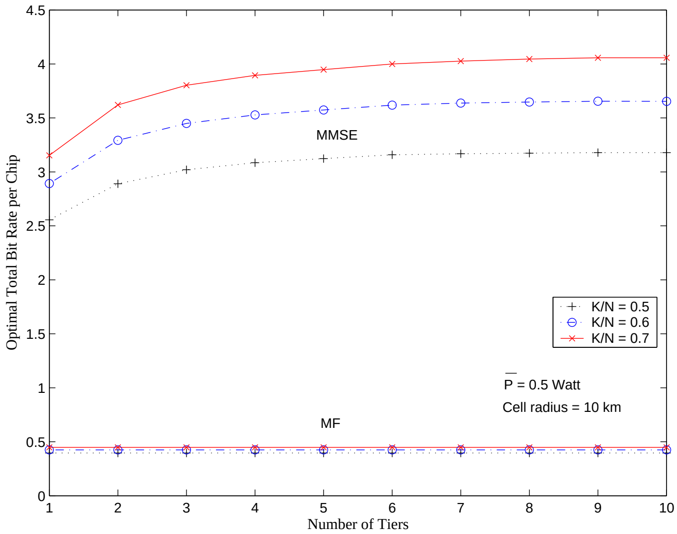

Fig. 2. Optimal total bit rate vs. the number of tiers for multiple loads with $\overline{P} = 0.5$ Watt and cell radius 10 km.

The effect of tiers on the optimal transmission rate is investigated as a function of the number of tiers, power budget, cell radius and mobile load. As a test bed, a DS-CDMA radio channel transmitting in frequency 900 MHz (common for GSM and W-CDMA) is considered. The channel fading is subject to exponential path loss with a path loss parameter $n = 3$ (typical for outdoor cellular environments), Lognormal shadow fading with a standard deviation of 8 dB, and Rayleigh fading with mean one. The AWGN power spectral density is fixed to 10^{-15} Watt, the peak transmission power is fixed to 2 Watt, and the required outage probability is 0.001. A cell of a given radius is partitioned into J ‘equiwidth’ tiers, each containing the same number of mobiles and having the same utility function. The utility function is the one defined in (24). That is, the rate of reliable transmitted symbols in an MF or an MMSE DS-CDMA channel.

Figures 1–3 depict the optimal total bit rate vs. the number of tiers along three different dimensions for

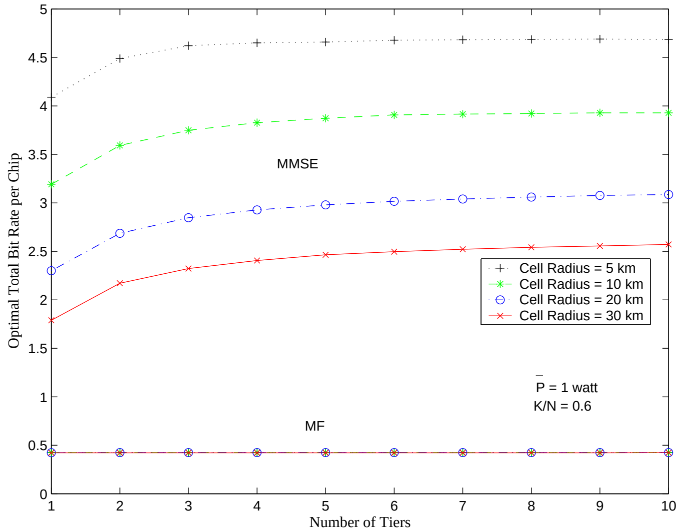

Fig. 3. Optimal total bit rate vs. the number of tiers for multiple cell radii with $\overline{P} = 1.0$ Watt and $\alpha = 0.6$.

both type of receivers, the MF and the MMSE. Figure 1 compares the utilities resulting from different power budgets, Figure 2 compares the utilities resulting from different loads and Figure 3 compares the utilities resulting from different cell radii.

The most striking observation is that an MF DS-CDMA channel has no noticeable benefit from any control parameter. The optimal total transmission rate remains flat around 0.4 symbols/chip, which is substantially below the optimal rates with the MMSE multiuser detector. The MMSE channel, on the other hand, does benefit from every control parameter. These phenomena are easily explained by studying the power budget constraints, (13) and (35), for the single class case.

For an MMSE channel with a single class, constraint (13) is reduced to

$$\gamma \leq \frac{\overline{P}\left(1 - \alpha(1-\epsilon)\frac{\gamma}{1+\gamma}\right)}{\alpha c}. \tag{38}$$

For an MF channel with a single class, constraint (33) is reduced to

$$\gamma \leq \frac{1}{\alpha(1-\epsilon) + \frac{\alpha c}{\overline{P}}}. \tag{39}$$

In light of the fact that in our test bed $c = O(10^{-4})$ and $\frac{\gamma}{1+\gamma} \approx 1$ for the optimal SIR target, the bound in (38) explains why an MMSE channel benefits from every control parameter, and the bound in (39) explains why an MF channel does not. The latter also explains why the MF channel has some benefit from increasing the load α (see Figure 2).

For the MMSE channel, it is observed that the total transmission rate increases with the number of tiers across all system parameters. However, the benefit from five tiers or more is marginal. The major benefit is due to two tiers, whereas $3-4$ tiers have a fair contribution. Note that this observation holds true for all cell radii, power budgets and mobile loads.

Another interesting observation for the MMSE channel is the optimal transmission rate distribution over the tiers. For instance, if the cell radius is 30 km, $\alpha = 0.6$ and the power budget is 1 Watt, then for the case with 10 tiers, $\boldsymbol{\gamma}^* = (47.9, 37.7, 31.7, 27.5, 24.3, 21.5, 19.5, 17.6, 15.8, 14.3)$ dB with a total rate of 2.57 symbols per chip. For the case with 3 tiers, $\boldsymbol{\gamma}^* = (32.2, 22.0, 15.7)$ dB with a total rate of 2.32 symbols per chip. As expected, the optimal rate allocation in an MMSE CDMA channel exploits good channels, and the closer the tier is, the larger is its allocated rate.

VII. Conclusion

The asymptotically optimal slow-rate-fast-power control policy was derived for a DS-CDMA time varying fading channel with multiple user classes, random spreading codes and a receiver using either a conventional MF or a linear MMSE multiuser detector.

The two problems were rigorously presented as constrained optimization programs, which were explicitly solved in a closed-form given by a function of a single Lagrangian multiplier. The key features of the optimal power and rate control are:

- The optimal combined power and rate control problems can be decomposed into two separate optimization problems.
- For every feasible rate vector, the optimal power control for each communication channel is a *threshold cut-off policy* that inverts the controlled channel gain if the gain is above a given threshold. Otherwise, it cuts off the power. The channel gain inversion is done so as to attain the optimal SIR targets determined by the optimal required rates. The cut-off threshold is pre-determined from the channel fading distribution.
- The optimal rates are given by a closed-form function of a single Lagrange multiplier. The optimal multiplier is determined by J independent fast bisection searches for a zero, where each search is done in a single-valued function and J is the number of user classes.
- For an MF CDMA channel and a total rate utility function, the optimal policy is completely specified in a closed form.
- In the case where MAI shrinks to zero, the problem reduces to a DS-SS Gaussian channel with a single user. From [13], $R_j^*(\gamma)$ in (7) (MMSE) and (30) (MF) reduces into $R_j^*(\gamma) = \gamma_j \sigma^2$, and both problems are unified.

The optimal control policy was also demonstrated by an application, where the transmission power is adapted to the channel gain variations, and the transmission rates are adapted to the tier containing the mobile. The effect of the number of tiers on the optimal transmission rate was presented in a cellular network with Lognormal and Rayleigh fading.

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