Burst Segmentation Benefit in Optical Switching

Zvi Rosberg, Hai Le Vu, *Member, IEEE*, and Moshe Zukerman, *Senior Member, IEEE*

***Abstract*—In this letter, we derive the asymptotic behavior of the ratio between the blocking probabilities of burst segmentation (BS) and just-enough-time (JET) policies in an optical burst switch. It is shown that if the ratio between the offered load and the number of wavelengths, k, is fixed and equals c as the number of wavelengths approaches infinity, the ratio between the blocking probabilities of BS and JET approaches 1 if $c > 1$; 0.5 if $c = 1$; and is $O(1/k)$ with a constant $c/(1-c)^2$ if $c < 1$.**

***Index Terms*—Asymptotic relation, blocking, burst segmentation, just-enough-time (JET), optical burst switching.**

I. Introduction

OPTICAL burst switching (OBS) (see [1]–[4] and [5]) has been proposed as an efficient switching technique to exploit the terabit bandwidth of wavelength-division multiplexing (WDM) transmission technology. In OBS, IP packets with a common destination arriving at the same ingress node are aggregated into large bursts, switched and routed as one unit. Having only one header associated with each burst, the header processing per transmitted bit is reduced, and the switch fabric can be reconfigured on a longer timescale. The control packet precedes the burst payload and attempts to reserve switching and transmission resources at each switch and output port along the route.

Note that optical switches based on semiconductor optical amplifiers (SOAs) achieve reconfiguration time in the order of a few hundred picoseconds, and electro-absorption modulator based devices are capable of reconfiguration time in the order of a few picoseconds [6]. With support from these technologies, OBS is likely to become a feasible switching technique in the near future.

A key feature in OBS is that the header precedes the payload by an offset time and is usually transmitted on a dedicated signalling wavelength. The payload follows the header without waiting for acknowledgment. At every switch, if the requested resources are available, the burst is transparently switched to its next hop; otherwise, the burst is blocked and some fraction (possibly all) of the data is lost.

We consider an output line of a given switch with k wavelengths and assume that an incoming burst can be transmitted on any available one. It models a switch with either full wavelength conversion or with no wavelength conversion but with k fibers in its outgoing link. Several OBS reservation protocols have been proposed by researchers. Under just-enough-time (JET) [1], the control packet contains the burst length and requests link bandwidth from the predetermined time offset and for the duration of the burst transmission. To reduce packet loss, there have been various proposals for burst segmentation (BS) [3] and [4]. In our version of BS (based on [3]), in case of contention, the control packet reserves capacity from the first instant a wavelength becomes available. The initial portion of the burst, which is not served before a wavelength becomes free, is dumped. Its remainder is transmitted successfully as a truncated burst.

The reader should be aware of the following two BS overhead types. One is due to losses occur during burst switching time and the other is due to the fact that the boundary of the truncated burst may not fall exactly at IP packet boundaries, in which case the IP packets remainders are lost. In this letter, we neglect these two effects. Notice that the second effect is negligible when the IP packets are small relative to the burst size which is a valid assumption.

In this letter we derive the asymptotic ratio between the blocking probabilities of BS and JET protocols as the number of wavelengths supported by the optical switch is large while maintaining the traffic per wavelength fixed. This limiting case is important and relevant given the growth in traffic demand and the growth in number of wavelength per link we have witnessed in the last 15 years. In Section II we derive the asymptotic relations and in Section III we present our conclusions.

II. Asymptotic Relations

Since OBS is a bufferless system, the $M/M/k/k$ queueing model is used in [1], [2] to express the blocking probability using the JET protocol. This model assumes that the burst arrival process at a given output port of an optical burst switch is a Poisson process with rate λ, burst transmission time is exponentially distributed with mean $1/\mu$, the number of wavelengths on the output fiber is k and there is no extra waiting buffers. In this case, the burst blocking probability is given by the following Erlang B formula:

$$P_B(k,A) = \frac{(A^k/k!)}{\sum_{m=0}^{k}(A^m/m!)} \tag{1}$$

where $A = \lambda/\mu$ is the offered load. Due to the insensitivity of the Erlang B formula to the service time distribution, the burst duration distribution can be relaxed to be a general distribution. Note that the packet loss probability is the same as the burst loss probability [3].

The BS policy cannot be modeled by the $M/M/k/k$ queueing model since blocked bursts are segmented and may

Manuscript received September 2, 2002. The associate editor coordinating the review of this letter and approving it for publication was Prof. J. Evans. This work was supported by the Australian Research Council.

Z. Rosberg was a visitor with the ARC Special Research Center for Ultra-Broadband Information Networks, University of Melbourne, Melbourne, Australia. He is with the Department of Communication Systems Engineering, Ben Gurion University, Beer-Sheva, 84105, Israel (e-mail: rosberg@bgumail.bgu.ac.il).

H. L. Vu and M. Zukerman are with the ARC Special Research Center for Ultra-Broadband Information Networks, Department of Electrical and Electronic Engineering, The University of Melbourne, Melbourne, Vic. 3010, Australia (e-mail: h.vu@ee.mu.oz.au; m.zukerman@ee.mu.oz.au).

Digital Object Identifier 10.1109/LCOMM.2003.809998

partially be rejected under this policy [3]. Nevertheless, the following simple formula for the packet blocking probability, when the packet size is negligible relative to the burst size, has been derived in [7]

$$P_{BS}(k,A)=\frac{E[L]}{A} \tag{2}$$

where $E[L]$ is the mean loss rate given by

$$E[L]=\sum_{j=1}^{\infty} j\cdot P(k+j). \tag{3}$$

Here $P(k+j)$ is the probability that $(k+j)$ servers are busy in an $M/G/\infty$ model, which is given by the Poisson probability function

$$P(k+j)=\frac{e^{-A}A^{k+j}}{(k+j)!}, \qquad j=1,2,\ldots$$

For the packet blocking probability whereby the packet size is not negligible relative to the burst size the reader is referred to [3].

It is intuitively clear (and proven below) that BS has lower packet blocking probability than JET. However, a simple expression for the ratio between these blocking probabilities is not available. Although a numerical procedure is derived in [7], it cannot be used for large practical values of k due to computational limitations. The asymptotic expression derived in this letter facilitates the computation for large k values which are relevant according to current technology developments and trends.

Define $R(k)=P_{BS}(k,A)/P_B(k,A)$. The function $R(k)$ presents the relative benefit of BS over JET. We explore the limit of $R(k)$ as $k\to\infty$ where the ratio $A/k=c$ for arbitrary constant c, namely, where the offered load per channel is fixed.

The first step toward this end is to represent $E[L]$ by an expression that facilitates the derivation of $R(k)$ as $k\to\infty$. Let X_A be a Poisson random variable with mean A, $[Z]^+$ equals Z if $Z>0$ and 0 otherwise, and $[Z]^-$ equals Z if $Z\le 0$ and 0,otherwise. Clearly,

$$X_A-k=[X_A-k]^+ + [X_A-k]^-. \tag{4}$$

From (3) it follows that $E[L]=E[X_A-k]^+$ and by a straightforward computation we obtain

$$\begin{aligned}-E[X_A-k]^- &= \sum_{i=0}^{k}(k-i)\Pr(X_A=i)\\ &=\Pr(X_A\le k)\sum_{i=0}^{k}(k-i)\frac{\Pr(X_A=i)}{\Pr(X_A\le k)}\\ &=\Pr(X_A\le k)\,(k-A(1-P_B(k,A)))\end{aligned} \tag{5}$$

where $A(1-P_B(k,A))$ is the number of busy servers in the Erlang loss system $M/M/k/k$.

Taking the expectations in both sides of (4) and substituting (2) and (5) yields

$$P_{BS}(k,A)=\Pr(X_A>k)\left(1-\frac{1}{c}\right)+\Pr(X_A\le k)P_B(k,A). \tag{6}$$

Thus, from the definition of $R(k)$ and (6)

$$R(k)=\Pr(X_A\le k)+\Pr(X_A>k)\,\frac{1-\frac{1}{c}}{P_B(k,A)}. \tag{7}$$

We are now ready to show that the packet blocking probability of BS is always smaller than that of JET.

Theorem 1: For every offered load A and number of wavelengths k, $P_{BS}(k,A)<P_B(k,A)$.

Proof: From (6), the Theorem assertion holds true if and only if $1-(1/c)<P_B(k,A)$. Since $c=(A/k)$, the assertion holds true if and only if

$$k>A(1-P_B(k,A))=E[N(k,A)]. \tag{8}$$

Since the expected number of busy servers in $M/M/M/k/k$ is strictly smaller than k, the Theorem assertion follows. ■

Next we utilize the following asymptotic relations from [8]

$$P_B(k,A)\sim\frac{\phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)}{\sqrt{c\cdot k}\,\Phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)} \tag{9}$$

$$\Pr(X_A\le k)\sim\Phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right) \tag{10}$$

where ϕ is the density and Φ is the cdf of a standard normal distribution (with mean 0 and variance 1), and $f(k)\sim g(k)$ means that $f(k)/g(k)\to 1$ as $k\to\infty$.

The following Lemma provides the asymptotic relations for each individual function of $R(k)$ as given by (7).

Lemma 1: As $k\to\infty$, the following limits apply for every offered load A and constant c:

i) If $c<1$, $\Pr(X_A\le k)\to 1$ and $P_B(k,A)\to 0$;
ii) If $c=1$, $\Pr(X_A\le k)\to 0.5$ and $P_B(k,A)\to 0$;
iii) If $c>1$, $\Pr(X_A\le k)\to 0$ and $P_B(k,A)\to 1-(1/c)$.

Proof: The limits of $\Pr(X_A\le k)$ directly follow from (10). The limits of $P_B(k,A)$ are derived from (9) as follows.

For $c<1$ and $c=1$, the cdf $\Phi((1-c/\sqrt{c})\sqrt{k})$ in the denominator of (9) converges to 1 and 0.5, respectively, as $k\to\infty$. Therefore, the denominator of (9) converges to ∞ while the numerator is bounded. Thus, for cases (i) and (ii), $P_B(k,A)\to 0$ as $k\to\infty$.

For $c>1$, the limits of both $\phi((1-c/\sqrt{c})\sqrt{k})$ and $\Phi((1-c/\sqrt{c})\sqrt{k})$ in (9) are zero, hence we use L'Hopital's rule to obtain the limit of their ratio. Since the ratio of their derivatives with respect to $(1-c/\sqrt{c})\sqrt{k}$ equals

$$\frac{\frac{c-1}{\sqrt{c}}\sqrt{k}\phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)}{\phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)}=\frac{c-1}{\sqrt{c}}\sqrt{k}$$

the limit of the right-hand side of (9) equals

$$\frac{\frac{c-1}{\sqrt{c}}\sqrt{k}}{\sqrt{c\cdot k}}=1-\frac{1}{c}$$

which completes the proof of case (iii). ■

Finally, the following Theorem provides the asymptotic relation of $R(k)$, i.e., of the relative benefit of BS over JET.

Theorem 2: For every offered load A and constant c:

i) If $c<1$ then $R(k)\sim c/(1-c)^2k$, i.e., $R(k)=O(1/k)$ with a constant $c/(1-c)^2$;
ii) If $c=1$ then $R(k)\to 0.5$ as $k\to\infty$;
iii) If $c>1$ then $R(k)\to 1$ as $k\to\infty$.

Proof: Cases (ii) and (iii) are direct consequences of Lemma 1 and (7).

For case (i) observe that (9), (10), and (7) imply that $R(k) \sim f(k)$, where

$$f(k)=\Phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)\left(1-\frac{(1-c)}{\sqrt{c}}\frac{\left(1-\Phi\left(\frac{(1-c)}{\sqrt{c}}\sqrt{k}\right)\right)}{\phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)}\sqrt{k}\right). \tag{11}$$

To obtain the asymptotic relation of $R(k)$ we derive two sequences, $l(k)$ and $u(k)$, which asymptotically bound its asymptotic relation $f(k)$ from below and above, respectively. To this end, we derive tight upper and lower bounds to the Gaussian distribution tail $1-\Phi(x)$ which is part of (11).

Since $\phi(x)$ does not have a closed form integral we bound it by the following two functions, $L(x)$ and $U(x)$, that do have closed form integrals and are approaching $\phi(x)$ from below and above, respectively, as $x \to \infty$:

$$L(x)=\phi(x)\left(1-\frac{3}{x^4}\right); \quad U(x)=\phi(x)\left(1+\frac{15}{x^6}\right). \tag{12}$$

By integration it is easy to show that

$$\phi(x)\left(\frac{1}{x}-\frac{1}{x^3}\right) < 1-\Phi(x) < \phi(x)\left(\frac{1}{x}-\frac{1}{x^3}+\frac{3}{x^5}\right). \tag{13}$$

Clearly, the bounds are tight only for large values of x. Introducing the upper and lower bounds from (13) with $x=(1-c/\sqrt{c})\sqrt{k}$ into $f(k)$, see (11), we obtain after basic algebraic operations,

$$f(k) < \Phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)\frac{c}{(1-c)^2 k} < \frac{c}{(1-c)^2 k} \stackrel{\text{def}}{=} u(k). \tag{14}$$

$$f(k) > u(k) - \Phi\left(\frac{1-c}{\sqrt{c}}\sqrt{k}\right)\frac{3c^2}{(1-c)^4\, k^2} \stackrel{\text{def}}{=} l(k). \tag{15}$$

Thus, from (14), $R(k)$ is asymptotically bounded above by the sequence $u(k)$. Moreover, from (15) the ratio $l(k)/u(k) \to 1$ as $k \to \infty$, which implies case (i) of the Theorem. ∎

The asymptotic relations above are verified against numerical computations of $R(k)$ using the procedure from [7]. The results are depicted in Figs. 1 and 2, and comply with our analytical results. Due to numerical insatiability in the computation of $R(k)$ for large k, the verification is limited only to the values presented in the figures. Overcoming this limitation signifies the importance of our asymptotic result.

III. Conclusion

We have analytically expressed the limit of the ratio between the blocking probabilities of BS and JET policies in a single optical burst switch as the capacity and the offered load increase at the same rate. We have shown that for the interesting case where $c=(A/k)<1$, the asymptotic value of this ratio is $O(1/k)$ with a constant $c/(1-c)^2$. That is, BS offers a factor of $((1-c)^2/c)\ k$ improvement in blocking probability value, which can be more than two orders of magnitude for foreseeable dense WDM systems. For instance, if $c=0.9$ and $k=1000$, the blocking probability of BS is 11 times lower than the blocking probability of JET, and it is 111 times lower for $k=10\,000$. Moreover, the lower is the offered load with respect to the service capacity, the greater is the relative improvement of BS. For example, if $c=0.5$, the improvement factor is $0.5\times k$, and if $c=0.3$, it is $1.63\times k$.

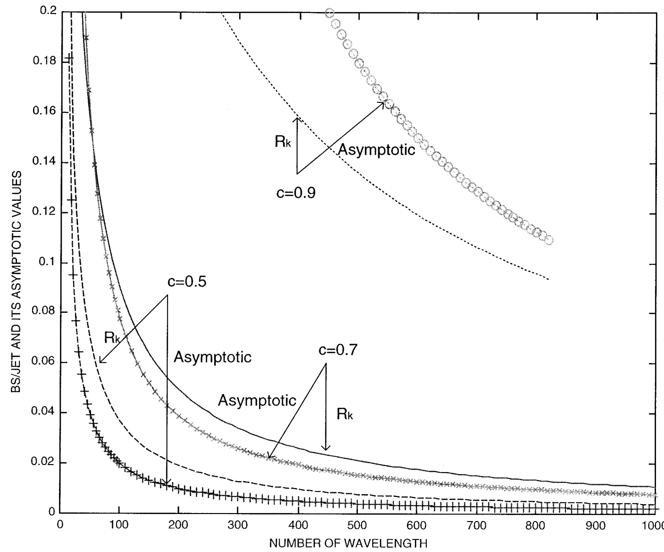

Fig. 1. The functions $R(k)$ and $u(k)$ for various $c<1$.

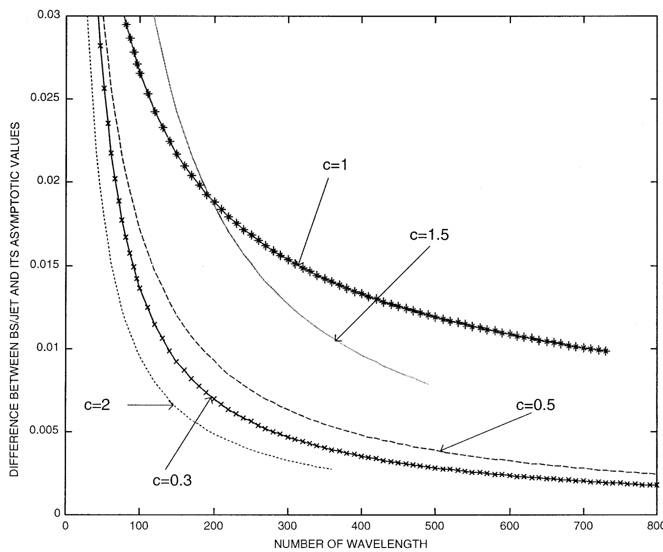

Fig. 2. The difference $R(k)-u(k)$ for various c.

References

[1] C. Qiao, "Labeled optical burst switching for IP-over-WDM integration," *IEEE Commun. Mag.*, pp. 104–114, Sept. 2000.

[2] K. Dolzer, C. Gauger, J. Spath, and S. Bodamer, "Evaluation of reservation mechanisms for optical burst switching," *AEU Int. J. Electron. Commun.*, vol. 55, no. 1, 2001.

[3] A. Detti, V. Eramo, and M. Listanti, "Performance evaluation of a new technique for IP support in a WDM optical network: Optical composite burst switching (OCBS)," *J. Lightwave Technol.*, vol. 20, no. 2, pp. 154–165, Feb. 2002.

[4] V. M. Vokkarane, J. P. Jue, and S. Sitaraman, "Burst segmentation: An approach for reducing packet loss in optical burst switched networks," in *Proc. ICC'02*, vol. 5, Apr. 2002, pp. 2673–2677.

[5] J. Y. Wei, J. L. Pastor, R. S. Ramamurthy, and Y. Tsai, "Just-in-time optical burst switching for multi-wavelength networks," in *Proc. 5th Int. Conf. on Broadband Commun. (BC'99)*, 1999, pp. 339–352.

[6] L. Rau *et al.*, "Two-hop all-optical label swapping with varaible length 80 Gb/s packets and 10 Gb/s labels using nonlinear fiber wavelength converters, unicast/multicast output and a single EAM for 80- to 10 Gb/s packet multiplexing," in *Proc. OFC 2002*, Anaheim, CA, Mar. 2002, pp. DF2 1–DF2 3.

[7] M. Neuts, Z. Rosberg, H. L. Vu, J. White, and M. Zukerman, "Performance analysis of optical composite burst switching," *IEEE Commun. Lett.*, vol. 6, pp. 346–348, Aug. 2002.

[8] D. L. Jagerman, "Some properties of the erlang loss function," *Bell Syst. Tech. J.*, vol. 53, pp. 525–551, 1974.